SAMPLE TEST ANSWERS

1. \[ \frac{10x^3+1}{x^2(x^2+9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+B}{x^2+9} = \frac{1/9}{x^2} + \frac{10x-1/9}{x^2+9} \]

2. \[ \frac{x-1}{(x+1)(x+2)} = -\frac{2}{x+1} + \frac{3}{x+2} \]
   \[ \int \frac{x-1}{(x+1)(x+2)} dx = -2 \ln |x+1| + 3 \ln |x+2| + C \]

3. \((Ae^t + Be^{-t})'' = Ae^t + Be^{-t}.\]

4. \[ \mu = e^{-2x^2}, e^{-2x^2}y = \frac{1}{2} \int e^{-2x^2} xdx = -\frac{1}{8} e^{-2x^2} + C, \]
   \[ y(0) = -\frac{1}{8} + C = 1 \text{ gives } C = \frac{9}{8}, \text{ so } y = -1 + \frac{9}{8} e^{2x^2}. \]
   Alternative: separation of variables.

5. \[ \frac{dy}{y^2-y} = \frac{dx}{\sin x}, \int \left( \frac{1}{y-1} - \frac{1}{y} \right) dy = \int \csc x dx, \]
   \[ \ln |y-1| - \ln |y| = -\ln |\csc x + \cot x| + C = \ln |\tan(\frac{x}{2})| + C, \]
   \[ 1 - \frac{1}{y} = A\tan(\frac{x}{2}), \quad y = \frac{1}{1-A\tan(\frac{x}{2})}. \]

6. Let \(A(t)\) be the amount of salt (in pounds) after \(t\) minutes.
   The rate at which salt enters the tank is \(2 \times 5 = 10\) (lb/min).
   Assuming that the volume of brine in the tank stays constant at 100 gal, the rate at which salt exits is \(\frac{A(t)}{100} \times 5 = \frac{A(t)}{20}\) (lb/min).
   Hence \(A' = 10 - A/20, A(0) = 4.\) This gives \(A = 200 + Ce^{-t/20},\)
   \(C = -196.\) So \(A(10) = 200 - 196e^{-5}.\)

7. Let \(P(t)\) be the population (in thousands), where \(t\) is years since 1998. Assuming exponential growth, \(P(t) = 10e^{kt}\) and \(P(5) = 12.\) Hence \(10e^{5k} = 12\) or \(k = .2\ln(1.2).\) This gives
   \[ t_{\text{double}} = \frac{\ln 2}{k} = \frac{5\ln 2}{\ln 1.2}. \]