TEST 2 ANSWERS

1. \( \int_{-1}^{1} (4 - (3 - x))dx + \int_{1}^{3} (4 - (x + 1))dx = \int_{-1}^{3} (2 - |x - 1|)dx (= 4) \)

2. \( \int_{2}^{4} ((y - 1) - (3 - y))dy = 2 \int_{2}^{4} (y - 2)dy (= 4) \)

3. \( \pi \int_{1}^{2} ((x - 1)^2 - (x - 1)^4)dx = \pi \int_{0}^{1} (x^2 - x^4)dx (= \frac{2\pi}{15}) \)

4. \( \pi \int_{0}^{1} ((\sqrt{x} + 1)^2 - (x + 1)^2)dx (= \frac{\pi}{2}) \)

5. Since \( 100 = 2k \), \( k = 500 \) N/m. So \( W = 500 \cdot 0.8^2 = 160 \) J.

6. The layer of water at level \( h \) has dimensions \( \frac{4}{3} \times 6 \times dh \) m\(^3\). This layer has weight \( 8\rho dh \) N (\( \rho \) is the weight density) and must be lifted a distance \( 3 - h \). The total work is therefore
   \[
   W = \int_{0}^{2} (3 - h)8\rho dh = 8\rho \int_{0}^{2} (3h - h^2)dh (= 261600 \) J.
   
7. Using integration by parts twice:
   \[
   \int x^2 e^x dx \quad u = x^2, v = e^x \\
   = x^2 e^x - 2 \int x e^x dx \\
   \quad u = x, v = e^x \\
   = x^2 e^x - 2xe^x + 2 \int e^x dx \\
   = x^2 e^x - 2xe^x + 2e^x + C.
   
   Check by differentiation.

8. Integration by parts with \( u = \arctan x, v = x \):
   \[
   \int_{0}^{1} \arctan x dx = x \arctan x \bigg|_{0}^{1} - \int_{0}^{1} x \frac{1}{1+x^2} \, dx \\
   = \frac{\pi}{4} - \frac{1}{2} \ln(x^2 + 1) \bigg|_{0}^{1} \\
   = \frac{\pi}{4} - \frac{1}{2} \ln 2.
   
9. Substitution \( x = \tan \theta, \) \( dx = \sec^2 \theta d\theta \), gives \( 1 + x^2 = \sec^2 \theta \) and so
   \[
   \int \frac{1}{(1 + x^2)} \, dx = \int \cos^2 \theta \, d\theta = \frac{1}{2} \int (1 + \cos(2\theta)) \, d\theta = \frac{1}{2} \theta + \frac{1}{4} \sin(2\theta) + C.
   
   Switching back to \( x \): \( \frac{1}{2} \arctan x + \frac{1}{2} \frac{x}{1+x^2} + C \). Check by differentiation.