2. If $y = C_1e^{2x} + C_2xe^{2x}$ is the general solution of a differential equation, then the order of the equation is ________, and a solution to the differential equation that satisfies the initial conditions $y(0) = 1, y'(0) = 4$ is given by $y =$ ________.

3. The graph of a differentiable function $y = y(x)$ passes through the point $(0, 1)$ and at every point $P(x, y)$ on the graph the tangent line is perpendicular to the line through P and the origin. Find an initial-value problem whose solution is $y(x)$.

4. A glass of ice water with a temperature of 36°F is placed in a room with a constant temperature of 68°F. Assuming that Newton's Law of Cooling applies, find an initial-value problem whose solution is the temperature of water t minutes after it is placed in the room. [*Note:* The differential equation will involve a constant of proportionality.]

EXERCISE SET 8.1

1. Confirm that $y = 3e^{x^3}$ is a solution of the initial-value problem $y' = 3x^2y, y(0) = 3$.

2. Confirm that $y = \frac{1}{4}x^4 + 2\cos x + 1$ is a solution of the initial-value problem $y' = x^3 - 2\sin x, y(0) = 3$.

3–4 State the order of the differential equation, and confirm that the functions in the given family are solutions.

3. (a) $(1 + x)\dfrac{dy}{dx} = y;\ \ y = c(1 + x)$

(b) $y'' + y = 0;\ \ y = c_1 \sin t + c_2 \cos t$

4. (a) $2\dfrac{dy}{dx} + y = x - 1;\ \ y = ce^{-x/2} + x - 3$

(b) $y'' - y = 0;\ \ y = c_1e^t + c_2e^{-t}$

5–8 True–False Determine whether the statement is true or false. Explain your answer.

5. The equation

$$\left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx} + 2y$$

is an example of a second-order differential equation.

6. The differential equation

$$\frac{dy}{dx} = 2y + 1$$

has a solution that is constant.

7. We expect the general solution of the differential equation

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - \frac{dy}{dx} + 4y = 0$$

to involve three arbitrary constants.

8. If every solution to a differential equation can be expressed in the form $y = Ae^{x+b}$ for some choice of constants A and b, then the differential equation must be of second order.

9–14 In each part, verify that the functions are solutions of the differential equation by substituting the functions into the equation.

9. $y'' + y' - 2y = 0$
(a) e^{-2x} and e^x
(b) $c_1e^{-2x} + c_2e^x$ (c_1, c_2 constants)

10. $y'' - y' - 6y = 0$
(a) e^{-2x} and e^{3x}
(b) $c_1e^{-2x} + c_2e^{3x}$ (c_1, c_2 constants)

11. $y'' - 4y' + 4y = 0$
(a) e^{2x} and xe^{2x}
(b) $c_1e^{2x} + c_2xe^{2x}$ (c_1, c_2 constants)

12. $y'' - 8y' + 16y = 0$
(a) e^{4x} and xe^{4x}
(b) $c_1e^{4x} + c_2xe^{4x}$ (c_1, c_2 constants)

13. $y'' + 4y = 0$
(a) $\sin 2x$ and $\cos 2x$
(b) $c_1 \sin 2x + c_2 \cos 2x$ (c_1, c_2 constants)

14. $y'' + 4y' + 13y = 0$
(a) $e^{-2x} \sin 3x$ and $e^{-2x} \cos 3x$
(b) $e^{-2x}(c_1 \sin 3x + c_2 \cos 3x)$ (c_1, c_2 constants)

15–20 Use the results of Exercises 9–14 to find a solution to the initial-value problem.

15. $y'' + y' - 2y = 0,\ y(0) = -1,\ y'(0) = -4$

16. $y'' - y' - 6y = 0,\ y(0) = 1,\ y'(0) = 8$

17. $y'' - 4y' + 4y = 0,\ y(0) = 2,\ y'(0) = 2$

18. $y'' - 8y' + 16y = 0,\ y(0) = 1,\ y'(0) = 1$

19. $y'' + 4y = 0,\ y(0) = 1,\ y'(0) = 2$

20. $y'' + 4y' + 13y = 0,\ y(0) = -1,\ y'(0) = -1$

21–26 Find a solution to the initial-value problem.

21. $y' + 4x = 2,\ y(0) = 3$

22. $y'' + 6x = 0,\ y(0) = 1,\ y'(0) = 2$

23. $y' - y^2 = 0,\ y(1) = 2$ [*Hint:* Assume the solution has an inverse function $x = x(y)$. Find, and solve, a differential equation that involves $x'(y)$.]

24. $y' = 1 + y^2,\ y(0) = 0$ (See Exercise 23.)

25. $x^2y' + 2xy = 0,\ y(1) = 2$ [*Hint:* Interpret the left-hand side of the equation as the derivative of a product of two functions.]

26. $xy' + y = e^x,\ y(1) = 1 + e$ (See Exercise 25.)

FOCUS ON CONCEPTS

27. (a) Suppose that a quantity $y = y(t)$ increases at a rate that is proportional to the square of the amount present, and suppose that at time $t = 0$, the amount present is y_0. Find an initial-value problem whose solution is $y(t)$.

(b) Suppose that a quantity $y = y(t)$ decreases at a rate that is proportional to the square of the amount present, and suppose that at a time $t = 0$, the amount present is y_0. Find an initial-value problem whose solution is $y(t)$.