✓ QUICK CHECK EXERCISES 9.6

(See page 563 for answers.)

1. What characterizes an *alternating* series?

2. (a) The series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$$

converges by the alternating series test since ________ and ________.

(b) If

$$S = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \quad \text{and} \quad s_9 = \sum_{k=1}^{9} \frac{(-1)^{k+1}}{k^2}$$

then $|S - s_9| <$ ________.

3. Classify each sequence as conditionally convergent, absolutely convergent, or divergent.

(a) $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{1}{k}$: ________

(b) $\sum_{k=1}^{\infty} (-1)^{k}\frac{3k-1}{9k+15}$: ________

(c) $\sum_{k=1}^{\infty} (-1)^{k}\frac{1}{k(k+2)}$: ________

(d) $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{1}{\sqrt[4]{k^3}}$: ________

4. Given that

$$\lim_{k\to+\infty} \frac{(k+1)^4/4^{k+1}}{k^4/4^k} = \lim_{k\to+\infty} \frac{\left(1+\frac{1}{k}\right)^4}{4} = \frac{1}{4}$$

is the series $\sum_{k=1}^{\infty} (-1)^k k^4/4^k$ conditionally convergent, absolutely convergent, or divergent?

EXERCISE SET 9.6 [C] CAS

1–2 Show that the series converges by confirming that it satisfies the hypotheses of the alternating series test (Theorem 9.6.1). ■

1. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2k+1}$ **2.** $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{k}{3^k}$

3–6 Determine whether the alternating series converges; justify your answer. ■

3. $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{k+1}{3k+1}$ **4.** $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{k+1}{\sqrt{k}+1}$

5. $\sum_{k=1}^{\infty} (-1)^{k+1}e^{-k}$ **6.** $\sum_{k=3}^{\infty} (-1)^{k}\frac{\ln k}{k}$

7–12 Use the ratio test for absolute convergence (Theorem 9.6.5) to determine whether the series converges or diverges. If the test is inconclusive, say so. ■

7. $\sum_{k=1}^{\infty} \left(-\frac{3}{5}\right)^k$ **8.** $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{2^k}{k!}$

9. $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{3^k}{k^2}$ **10.** $\sum_{k=1}^{\infty} (-1)^{k}\frac{k}{5^k}$

11. $\sum_{k=1}^{\infty} (-1)^{k}\frac{k^3}{e^k}$ **12.** $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{k^k}{k!}$

13–28 Classify each series as absolutely convergent, conditionally convergent, or divergent. ■

13. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{3k}$ **14.** $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{4/3}}$ **15.** $\sum_{k=1}^{\infty} \frac{(-4)^k}{k^2}$

16. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!}$ **17.** $\sum_{k=1}^{\infty} \frac{\cos k\pi}{k}$ **18.** $\sum_{k=3}^{\infty} \frac{(-1)^k \ln k}{k}$

19. $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{k+2}{k(k+3)}$ **20.** $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k^2}{k^3+1}$

21. $\sum_{k=1}^{\infty} \sin\frac{k\pi}{2}$ **22.** $\sum_{k=1}^{\infty} \frac{\sin k}{k^3}$

23. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k\ln k}$ **24.** $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k(k+1)}}$

25. $\sum_{k=2}^{\infty} \left(-\frac{1}{\ln k}\right)^k$ **26.** $\sum_{k=1}^{\infty} \frac{k\cos k\pi}{k^2+1}$

27. $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k!}{(2k-1)!}$ **28.** $\sum_{k=1}^{\infty} (-1)^{k+1}\frac{3^{2k-1}}{k^2+1}$

29–32 True–False Determine whether the statement is true or false. Explain your answer. ■

29. An alternating series is one whose terms alternate between even and odd.

30. If a series satisfies the hypothesis of the alternating series test, then the sequence of partial sums of the series oscillates between overestimates and underestimates for the sum of the series.

31. If a series converges, then either it converges absolutely or it converges conditionally.

32. If $\sum (u_k)^2$ converges, then $\sum u_k$ converges absolutely.