5. (a) If a function f has nth Taylor polynomial $p_n(x)$ about $x = x_0$, then the nth remainder $R_n(x)$ is defined by $R_n(x) =$ ______.

(b) Suppose that a function f can be differentiated five times on an interval containing the number $x_0 = 2$ andthat $|f^{(5)}(x)| \leq 20$ for all x in the interval. Then the fourth remainder satisfies $|R_4(x)| \leq$ ______ for all x in the interval.

EXERCISE SET 9.7 Graphing Utility

1–2 In each part, find the local quadratic approximation of f at $x = x_0$, and use that approximation to find the local linear approximation of f at x_0. Use a graphing utility to graph f and the two approximations on the same screen.

1. (a) $f(x) = e^{-x}$; $x_0 = 0$ (b) $f(x) = \cos x$; $x_0 = 0$

2. (a) $f(x) = \sin x$; $x_0 = \pi/2$ (b) $f(x) = \sqrt{x}$; $x_0 = 1$

3. (a) Find the local quadratic approximation of $\sqrt{x}$ at $x_0 = 1$.
(b) Use the result obtained in part (a) to approximate $\sqrt{1.1}$, and compare your approximation to that produced directly by your calculating utility. [*Note:* See Example 1 of Section 3.5.]

4. (a) Find the local quadratic approximation of $\cos x$ at $x_0 = 0$.
(b) Use the result obtained in part (a) to approximate $\cos 2^\circ$, and compare the approximation to that produced directly by your calculating utility.

5. Use an appropriate local quadratic approximation to approximate $\tan 61^\circ$, and compare the result to that produced directly by your calculating utility.

6. Use an appropriate local quadratic approximation to approximate $\sqrt{36.03}$, and compare the result to that produced directly by your calculating utility.

7–16 Find the Maclaurin polynomials of orders $n = 0, 1, 2, 3$, and 4, and then find the nth Maclaurin polynomials for the function in sigma notation.

7. e^{-x} **8.** e^{ax} **9.** $\cos \pi x$

10. $\sin \pi x$ **11.** $\ln(1 + x)$ **12.** $\dfrac{1}{1+x}$

13. $\cosh x$ **14.** $\sinh x$ **15.** $x \sin x$

16. xe^x

17–24 Find the Taylor polynomials of orders $n = 0, 1, 2, 3$, and 4 about $x = x_0$, and then find the nth Taylor polynomial for the function in sigma notation.

17. e^x; $x_0 = 1$ **18.** e^{-x}; $x_0 = \ln 2$

19. $\dfrac{1}{x}$; $x_0 = -1$ **20.** $\dfrac{1}{x+2}$; $x_0 = 3$

21. $\sin \pi x$; $x_0 = \dfrac{1}{2}$ **22.** $\cos x$; $x_0 = \dfrac{\pi}{2}$

23. $\ln x$; $x_0 = 1$ **24.** $\ln x$; $x_0 = e$

25. (a) Find the third Maclaurin polynomial for
$$f(x) = 1 + 2x - x^2 + x^3$$
(b) Find the third Taylor polynomial about $x = 1$ for
$$f(x) = 1 + 2(x-1) - (x-1)^2 + (x-1)^3$$

26. (a) Find the nth Maclaurin polynomial for
$$f(x) = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n$$
(b) Find the nth Taylor polynomial about $x = 1$ for
$$f(x) = c_0 + c_1(x-1) + c_2(x-1)^2 + \cdots + c_n(x-1)^n$$

27–30 Find the first four distinct Taylor polynomials about $x = x_0$, and use a graphing utility to graph the given function and the Taylor polynomials on the same screen.

27. $f(x) = e^{-2x}$; $x_0 = 0$ **28.** $f(x) = \sin x$; $x_0 = \pi/2$

29. $f(x) = \cos x$; $x_0 = \pi$ **30.** $\ln(x + 1)$; $x_0 = 0$

31–34 True–False Determine whether the statement is true or false. Explain your answer.

31. The equation of a tangent line to a differentiable function is a first-degree Taylor polynomial for that function.

32. The graph of a function f and the graph of its Maclaurin polynomial have a common y-intercept.

33. If $p_6(x)$ is the sixth-degree Taylor polynomial for a function f about $x = x_0$, then $p_6^{(4)}(x_0) = 4!f^{(4)}(x_0)$.

34. If $p_4(x)$ is the fourth-degree Maclaurin polynomial for e^x, then
$$|e^2 - p_4(2)| \leq \frac{9}{5!}$$

35–36 Use the method of Example 7 to approximate the given expression to the specified accuracy. Check your answer to that produced directly by your calculating utility.

35. $\sqrt{e}$; four decimal-place accuracy

36. $1/e$; three decimal-place accuracy

FOCUS ON CONCEPTS

37. Which of the functions graphed in the following figure is most likely to have $p(x) = 1 - x + 2x^2$ as its second-order Maclaurin polynomial? Explain your reasoning.