EXERCISE SET 9.8 Graphing Utility C CAS

1–10 Use sigma notation to write the Maclaurin series for the function.

1. e^{-x} 2. e^{ax} 3. $\cos \pi x$ 4. $\sin \pi x$
5. $\ln(1+x)$ 6. $\dfrac{1}{1+x}$ 7. $\cosh x$
8. $\sinh x$ 9. $x \sin x$ 10. xe^{x}

11–18 Use sigma notation to write the Taylor series about $x = x_0$ for the function.

11. e^x; $x_0 = 1$ 12. e^{-x}; $x_0 = \ln 2$
13. $\dfrac{1}{x}$; $x_0 = -1$ 14. $\dfrac{1}{x+2}$; $x_0 = 3$
15. $\sin \pi x$; $x_0 = \dfrac{1}{2}$ 16. $\cos x$; $x_0 = \dfrac{\pi}{2}$
17. $\ln x$; $x_0 = 1$ 18. $\ln x$; $x_0 = e$

19–22 Find the interval of convergence of the power series, and find a familiar function that is represented by the power series on that interval.

19. $1 - x + x^2 - x^3 + \cdots + (-1)^k x^k + \cdots$
20. $1 + x^2 + x^4 + \cdots + x^{2k} + \cdots$
21. $1 + (x-2) + (x-2)^2 + \cdots + (x-2)^k + \cdots$
22. $1 - (x+3) + (x+3)^2 - (x+3)^3 + \cdots + (-1)^k (x+3)^k | \ldots$

23. Suppose that the function f is represented by the power series

$$f(x) = 1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \cdots + (-1)^k \frac{x^k}{2^k} + \cdots$$

(a) Find the domain of f. (b) Find $f(0)$ and $f(1)$.

24. Suppose that the function f is represented by the power series

$$f(x) = 1 - \frac{x-5}{3} + \frac{(x-5)^2}{3^2} - \frac{(x-5)^3}{3^3} + \cdots$$

(a) Find the domain of f. (b) Find $f(3)$ and $f(6)$.

25–28 True–False Determine whether the statement is true or false. Explain your answer.

25. If a power series in x converges conditionally at $x = 3$, then the series converges if $|x| < 3$ and diverges if $|x| > 3$.
26. The ratio test is often useful to determine convergence at the endpoints of the interval of convergence of a power series.
27. The Maclaurin series for a polynomial function has radius of convergence $+\infty$.
28. The series $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!}$ converges if $|x| < 1$.

29–50 Find the radius of convergence and the interval of convergence.

29. $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k+1}$ 30. $\displaystyle\sum_{k=0}^{\infty} 3^k x^k$ 31. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k!}$

32. $\displaystyle\sum_{k=0}^{\infty} \frac{k!}{2^k} x^k$ 33. $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{k^2} x^k$ 34. $\displaystyle\sum_{k=2}^{\infty} \frac{x^k}{\ln k}$

35. $\displaystyle\sum_{k=1}^{\infty} \frac{x^k}{k(k+1)}$ 36. $\displaystyle\sum_{k=0}^{\infty} \frac{(-2)^k x^{k+1}}{k+1}$

37. $\displaystyle\sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^k}{\sqrt{k}}$ 38. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$

39. $\displaystyle\sum_{k=0}^{\infty} \frac{3^k}{k!} x^k$ 40. $\displaystyle\sum_{k=2}^{\infty} (-1)^{k+1} \frac{x^k}{k(\ln k)^2}$

41. $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{1+k^2}$ 42. $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$

43. $\displaystyle\sum_{k=0}^{\infty} \left(\frac{3}{4}\right)^k (x+5)^k$ 44. $\displaystyle\sum_{k=0}^{\infty} \frac{(x-3)^k}{2^k}$

45. $\displaystyle\sum_{k=1}^{\infty} (-1)^{k+1} \frac{(x+1)^k}{k}$ 46. $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{(x-4)^k}{(k+1)^2}$

47. $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{(x+1)^{2k+1}}{k^2+4}$ 48. $\displaystyle\sum_{k=1}^{\infty} \frac{(2k+1)!}{k^3} (x-2)^k$

49. $\displaystyle\sum_{k=0}^{\infty} \frac{\pi^k (x-1)^{2k}}{(2k+1)!}$ 50. $\displaystyle\sum_{k=0}^{\infty} \frac{(2x-3)^k}{4^{2k}}$

51. Use the root test to find the interval of convergence of

$$\sum_{k=2}^{\infty} \frac{x^k}{(\ln k)^k}$$

52. Find the domain of the function

$$f(x) = \sum_{k=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{(2k-2)!} x^k$$

53. Show that the series

$$1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots$$

is the Maclaurin series for the function

$$f(x) = \begin{cases} \cos \sqrt{x}, & x \geq 0 \\ \cosh \sqrt{-x}, & x < 0 \end{cases}$$

[*Hint:* Use the Maclaurin series for $\cos x$ and $\cosh x$ to obtain series for $\cos \sqrt{x}$, where $x \geq 0$, and $\cosh \sqrt{-x}$, where $x \leq 0$.]

FOCUS ON CONCEPTS

54. If a function f is represented by a power series on an interval, then the graphs of the partial sums can be used as approximations to the graph of f.
(a) Use a graphing utility to generate the graph of $1/(1-x)$ together with the graphs of the first four partial sums of its Maclaurin series over the interval $(-1, 1)$.

(cont.)