EXERCISE SET 8.2 Graphing Utility CAS

1–10 Solve the differential equation by separation of variables. Where reasonable, express the family of solutions as explicit functions of x.

1. $\dfrac{dy}{dx} = \dfrac{y}{x}$

2. $\dfrac{dy}{dx} = 2(1+y^2)x$

3. $\dfrac{\sqrt{1+x^2}}{1+y}\dfrac{dy}{dx} = -x$

4. $(1+x^4)\dfrac{dy}{dx} = \dfrac{x^3}{y}$

5. $(2+2y^2)y' = e^x y$

6. $y' = -xy$

7. $e^{-y}\sin x - y'\cos^2 x = 0$

8. $y' - (1+x)(1+y^2) = 0$

9. $\dfrac{dy}{dx} - \dfrac{y^2 - y}{\sin x} = 0$

10. $y - \dfrac{dy}{dx}\sec x = 0$

11–14 Solve the initial-value problem by separation of variables.

11. $y' = \dfrac{3x^2}{2y + \cos y}, \quad y(0) = \pi$

12. $y' - xe^y = 2e^y, \quad y(0) = 0$

13. $\dfrac{dy}{dt} = \dfrac{2t+1}{2y-2}, \quad y(0) = -1$

14. $y'\cosh^2 x - y\cosh 2x = 0, \quad y(0) = 3$

15. (a) Sketch some typical integral curves of the differential equation $y' = y/2x$.
(b) Find an equation for the integral curve that passes through the point $(2, 1)$.

16. (a) Sketch some typical integral curves of the differential equation $y' = -x/y$.
(b) Find an equation for the integral curve that passes through the point $(3, 4)$.

17–18 Solve the differential equation and then use a graphing utility to generate five integral curves for the equation.

17. $(x^2+4)\dfrac{dy}{dx} + xy = 0$

18. $(\cos y)y' = \cos x$

19–20 Solve the differential equation. If you have a CAS with implicit plotting capability, use the CAS to generate five integral curves for the equation.

19. $y' = \dfrac{x^2}{1-y^2}$

20. $y' = \dfrac{y}{1+y^2}$

21–24 True–False Determine whether the statement is true or false. Explain your answer.

21. Every differential equation of the form $y' = f(y)$ is separable.

22. A differential equation of the form

$$h(x)\frac{dy}{dx} = g(y)$$

is not separable.

23. If a radioactive element has a half-life of 1 minute, and if a container holds 32 g of the element at 1:00 P.M., then the amount remaining at 1:05 P.M. will be 1 g.

24. If a population is growing exponentially, then the time it takes the population to quadruple is independent of the size of the population.

25. Suppose that the initial condition in Example 1 had been $y(0) = 0$. Show that none of the solutions generated in Example 1 satisfy this initial condition, and then solve the initial-value problem

$$\frac{dy}{dx} = -4xy^2, \quad y(0) = 0$$

Why does the method of Example 1 fail to produce this particular solution?

26. Find all ordered pairs (x_0, y_0) such that if the initial condition in Example 1 is replaced by $y(x_0) = y_0$, the solution of the resulting initial-value problem is defined for all real numbers.

27. Find an equation of a curve with x-intercept 2 whose tangent line at any point (x, y) has slope xe^{-y}.

28. Use a graphing utility to generate a curve that passes through the point $(1, 1)$ and whose tangent line at (x, y) is perpendicular to the line through (x, y) with slope $-2y/(3x^2)$.

29. Suppose that an initial population of 10,000 bacteria grows exponentially at a rate of 2% per hour and that $y = y(t)$ is the number of bacteria present t hours later.
(a) Find an initial-value problem whose solution is $y(t)$.
(b) Find a formula for $y(t)$.
(c) How long does it take for the initial population of bacteria to double?
(d) How long does it take for the population of bacteria to reach 45,000?

30. A cell of the bacterium *E. coli* divides into two cells every 20 minutes when placed in a nutrient culture. Let $y = y(t)$ be the number of cells that are present t minutes after a single cell is placed in the culture. Assume that the growth of the bacteria is approximated by an exponential growth model.
(a) Find an initial-value problem whose solution is $y(t)$.
(b) Find a formula for $y(t)$.
(c) How many cells are present after 2 hours?
(d) How long does it take for the number of cells to reach 1,000,000?

31. Radon-222 is a radioactive gas with a half-life of 3.83 days. This gas is a health hazard because it tends to get trapped in the basements of houses, and many health officials suggest that homeowners seal their basements to prevent entry of the gas. Assume that 5.0×10^7 radon atoms are trapped in a basement at the time it is sealed and that $y(t)$ is the number of atoms present t days later.
(a) Find an initial-value problem whose solution is $y(t)$.
(b) Find a formula for $y(t)$.
(c) How many atoms will be present after 30 days?
(d) How long will it take for 90% of the original quantity of gas to decay?

32. Methylmercury is a toxic compound that can lead to neurological problems and an increased risk of heart disease. It takes about 50 days for the human body to eliminate 50% of a given quantity of the compound from the system. Suppose a test shows that there are 600 micrograms of