the accompanying figure. Use this slope field and geometric reasoning to find the integral curve that passes through the point (1, 2).

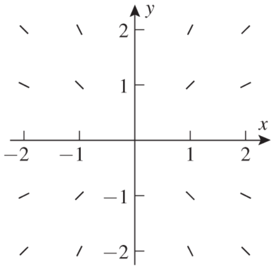

Figure Ex-2

3. When using Euler's Method on the initial-value problem $y' = f(x, y)$, $y(x_0) = y_0$, we obtain y_{n+1} from y_n, x_n, and Δx by means of the formula $y_{n+1} =$ ______.

4. Consider the initial-value problem $y' = y$, $y(0) = 1$.
   (a) Use Euler's Method with two steps to approximate $y(1)$.
   (b) What is the exact value of $y(1)$?

EXERCISE SET 8.3 Graphing Utility

1. Sketch the slope field for $y' = xy/4$ at the 25 gridpoints (x, y), where $x = -2, -1, \ldots, 2$ and $y = -2, -1, \ldots, 2$.

2. Sketch the slope field for $y' + y = 2$ at the 25 gridpoints (x, y), where $x = 0, 1, \ldots, 4$ and $y = 0, 1, \ldots, 4$.

3. A slope field for the differential equation $y' = 1 - y$ is shown in the accompanying figure. In each part, sketch the graph of the solution that satisfies the initial condition.
   (a) $y(0) = -1$ (b) $y(0) = 1$ (c) $y(0) = 2$

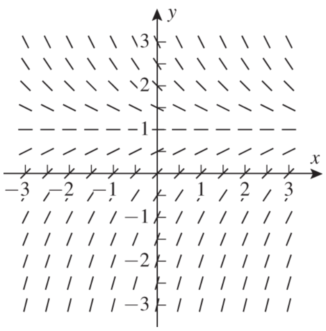

Figure Ex-3

4. Solve the initial-value problems in Exercise 3, and use a graphing utility to confirm that the integral curves for these solutions are consistent with the sketches you obtained from the slope field.

FOCUS ON CONCEPTS

5. Use the slope field in Exercise 3 to make a conjecture about the behavior of the solutions of $y' = 1 - y$ as $x \to +\infty$, and confirm your conjecture by examining the general solution of the equation.

6. In parts (a)–(f), match the differential equation with the slope field, and explain your reasoning.
   (a) $y' = 1/x$ (b) $y' = 1/y$
   (c) $y' = e^{-x^2}$ (d) $y' = y^2 - 1$
   (e) $y' = \dfrac{x + y}{x - y}$ (f) $y' = (\sin x)(\sin y)$

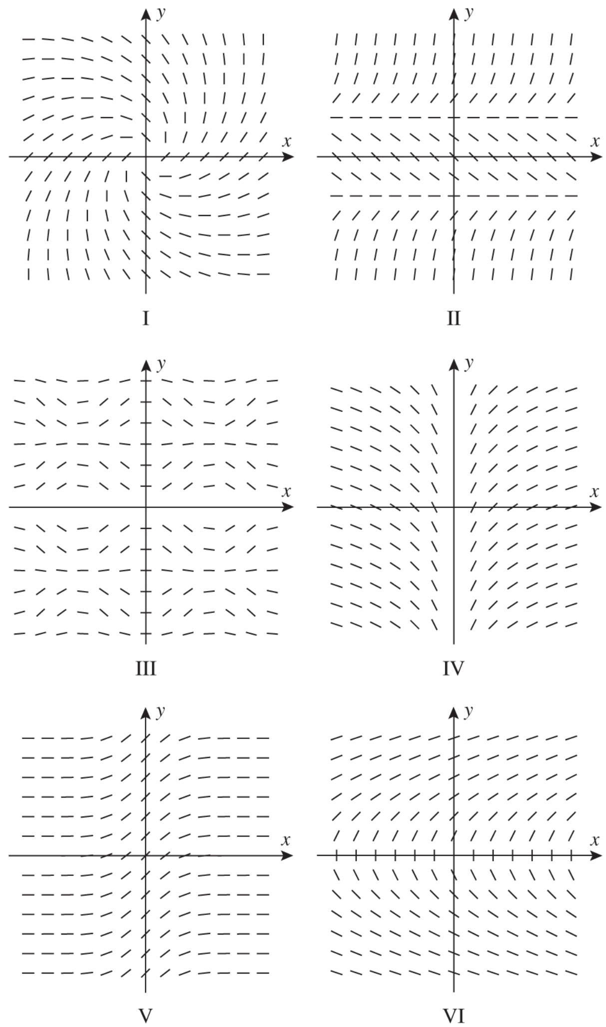

Figure Ex-6

7–10 Use Euler's Method with the given step size Δx or Δt to approximate the solution of the initial-value problem over the stated interval. Present your answer as a table and as a graph.

7. $dy/dx = \sqrt[3]{y}$, $y(0) = 1$, $0 \le x \le 4$, $\Delta x = 0.5$

8. $dy/dx = x - y^2$, $y(0) = 1$, $0 \le x \le 2$, $\Delta x = 0.25$

9. $dy/dt = \cos y,\ y(0) = 1,\ 0 \le t \le 2,\ \Delta t = 0.5$

10. $dy/dt = e^{-y},\ y(0) = 0,\ 0 \le t \le 1,\ \Delta t = 0.1$

11. Consider the initial-value problem

$$y' = \sin \pi t, \quad y(0) = 0$$

Use Euler's Method with five steps to approximate $y(1)$.

12–15 True–False Determine whether the statement is true or false. Explain your answer.

12. If the graph of $y = f(x)$ is an integral curve for a slope field, then so is any vertical translation of this graph.

13. Every integral curve for the slope field $dy/dx = e^{xy}$ is the graph of an increasing function of x.

14. Every integral curve for the slope field $dy/dx = e^{y}$ is concave up.

15. If $p(y)$ is a cubic polynomial in y, then the slope field $dy/dx = p(y)$ has an integral curve that is a horizontal line.

FOCUS ON CONCEPTS

16. (a) Show that the solution of the initial-value problem $y' = e^{-x^2}, y(0) = 0$ is

$$y(x) = \int_0^x e^{-t^2}\,dt$$

(b) Use Euler's Method with $\Delta x = 0.05$ to approximate the value of

$$y(1) = \int_0^1 e^{-t^2}\,dt$$

and compare the answer to that produced by a calculating utility with a numerical integration capability.

17. The accompanying figure shows a slope field for the differential equation $y' = -x/y$.

(a) Use the slope field to estimate $y\left(\frac{1}{2}\right)$ for the solution that satisfies the given initial condition $y(0) = 1$.

(b) Compare your estimate to the exact value of $y\left(\frac{1}{2}\right)$.

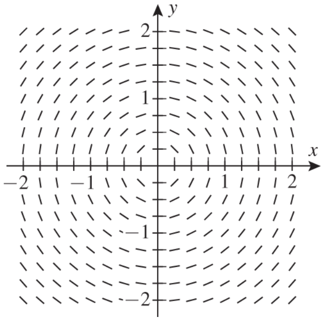

Figure Ex-17

18. Refer to slope field II in Quick Check Exercise 1.

(a) Does the slope field appear to have a horizontal line as an integral curve?

(b) Use the differential equation for the slope field to verify your answer to part (a).

19. Refer to the slope field in Exercise 3 and consider the integral curve through $(0, -1)$.

(a) Use the slope field to estimate where the integral curve intersects the x-axis.

(b) Compare your estimate in part (a) with the exact value of the x-intercept for the integral curve.

20. Consider the initial-value problem

$$\frac{dy}{dx} = \frac{\sqrt{y}}{2}, \quad y(0) = 1$$

(a) Use Euler's Method with step sizes of $\Delta x = 0.2$, 0.1, and 0.05 to obtain three approximations of $y(1)$.

(b) Find $y(1)$ exactly.

21. A slope field of the form $y' = f(y)$ is said to be ***autonomous***.

(a) Explain why the tangent segments along any horizontal line will be parallel for an autonomous slope field.

(b) The word *autonomous* means "independent." In what sense is an autonomous slope field independent?

(c) Suppose that $G(y)$ is an antiderivative of $1/[f(y)]$ and that C is a constant. Explain why any differentiable function defined implicitly by $G(y) - x = C$ will be a solution to the equation $y' = f(y)$.

22. (a) Solve the equation $y' = \sqrt{y}$ and show that every nonconstant solution has a graph that is everywhere concave up.

(b) Explain how the conclusion in part (a) may be obtained directly from the equation $y' = \sqrt{y}$ without solving.

23. (a) Find a slope field whose integral curve through $(1, 1)$ satisfies $xy^3 - x^2y = 0$ by differentiating this equation implicitly.

(b) Prove that if $y(x)$ is any integral curve of the slope field in part (a), then $x[y(x)]^3 - x^2y(x)$ will be a constant function.

(c) Find an equation that implicitly defines the integral curve through $(-1, -1)$ of the slope field in part (a).

24. (a) Find a slope field whose integral curve through $(0, 0)$ satisfies $xe^y + ye^x = 0$ by differentiating this equation implicitly.

(b) Prove that if $y(x)$ is any integral curve of the slope field in part (a), then $xe^{y(x)} + y(x)e^x$ will be a constant function.

(c) Find an equation that implicitly defines the integral curve through $(1, 1)$ of the slope field in part (a).

25. Consider the initial-value problem $y' = y$, $y(0) = 1$, and let y_n denote the approximation of $y(1)$ using Euler's Method with n steps.

(a) What would you conjecture is the exact value of $\lim_{n\to+\infty} y_n$? Explain your reasoning.

(b) Find an explicit formula for y_n and use it to verify your conjecture in part (a).

26. Writing Explain the connection between Euler's Method and the local linear approximation discussed in Section 3.5.

27. Writing Given a slope field, what features of an integral curve might be discussed from the slope field? Apply your ideas to the slope field in Exercise 3.