(verify). Thus, the speed $|v(t)|$ does not increase indefinitely, as in free fall; rather, because of the air resistance, it approaches a finite limiting speed v_τ given by

$$v_\tau = \left|-\frac{mg}{c}\right| = \frac{mg}{c} \qquad (18)$$

This is called the ***terminal speed*** of the object, and (17) is called its ***terminal velocity***.

REMARK Intuition suggests that near the limiting velocity, the velocity $v(t)$ changes very slowly; that is, $dv/dt \approx 0$. Thus, it should not be surprising that the limiting velocity can be obtained informally from (14) by setting $dv/dt = 0$ in the differential equation and solving for v. This yields

$$v = -\frac{mg}{c}$$

which agrees with (17).

✓ QUICK CHECK EXERCISES 8.4 (See page 511 for answers.)

1. Solve the first-order linear differential equation

$$\frac{dy}{dx} + p(x)y = q(x)$$

by completing the following steps:

Step 1. Calculate the integrating factor $\mu =$ ________.

Step 2. Multiply both sides of the equation by the integrating factor and express the result as

$$\frac{d}{dx}[\_\_\_\_\_\_\_\_] = \_\_\_\_\_\_\_\_$$

Step 3. Integrate both sides of the equation obtained in Step 2 and solve for $y =$ ________.

2. An integrating factor for

$$\frac{dy}{dx} + \frac{y}{x} = q(x)$$

is ________.

3. At time $t = 0$, a tank contains 30 oz of salt dissolved in 60 gal of water. Then brine containing 5 oz of salt per gallon of brine is allowed to enter the tank at a rate of 3 gal/min and the mixed solution is drained from the tank at the same rate. Give an initial-value problem satisfied by the amount of salt $y(t)$ in the tank at time t. Do not solve the problem.

EXERCISE SET 8.4 Graphing Utility

1–6 Solve the differential equation by the method of integrating factors.

1. $\dfrac{dy}{dx} + 4y = e^{-3x}$

2. $\dfrac{dy}{dx} + 2xy = x$

3. $y' + y = \cos(e^x)$

4. $2\dfrac{dy}{dx} + 4y = 1$

5. $(x^2 + 1)\dfrac{dy}{dx} + xy = 0$

6. $\dfrac{dy}{dx} + y + \dfrac{1}{1 - e^x} = 0$

7–10 Solve the initial-value problem.

7. $x\dfrac{dy}{dx} + y = x, \quad y(1) = 2$

8. $x\dfrac{dy}{dx} - y = x^2, \quad y(1) = -1$

9. $\dfrac{dy}{dx} - 2xy = 2x, \quad y(0) = 3$

10. $\dfrac{dy}{dt} + y = 2, \quad y(0) = 1$

11–14 True–False Determine whether the statement is true or false. Explain your answer.

11. If y_1 and y_2 are two solutions to a first-order linear differential equation, then $y = y_1 + y_2$ is also a solution.

12. If the first-order linear differential equation

$$\frac{dy}{dx} + p(x)y = q(x)$$

has a solution that is a constant function, then $q(x)$ is a constant multiple of $p(x)$.

13. In a mixing problem, we expect the concentration of the dissolved substance within the tank to approach a finite limit over time.

14. In our model for free-fall motion retarded by air resistance, the terminal velocity is proportional to the weight of the falling object.

15. A slope field for the differential equation $y' = 2y - x$ is shown in the accompanying figure on the next page. In each