as $n \to +\infty$, we obtain

$$L = \frac{1}{2}\left(L + \frac{2}{L}\right)$$

which can be rewritten as $L^2 = 2$. The negative solution of this equation is extraneous because $x_n > 0$ for all n, so $L = \sqrt{2}$. ◀

✓QUICK CHECK EXERCISES 9.1

(See page 524 for answers.)

1. Consider the sequence 4, 6, 8, 10, 12, ...
   (a) If $\{a_n\}_{n=1}^{+\infty}$ denotes this sequence, then $a_1 =$ ______, $a_4 =$ ______, and $a_7 =$ ______. The general term is $a_n =$ ______.
   (b) If $\{b_n\}_{n=0}^{+\infty}$ denotes this sequence, then $b_0 =$ ______, $b_4 =$ ______, and $b_8 =$ ______. The general term is $b_n =$ ______.
2. What does it mean to say that a sequence $\{a_n\}$ converges?
3. Consider sequences $\{a_n\}$ and $\{b_n\}$, where $a_n \to 2$ as $n \to +\infty$ and $b_n = (-1)^n$. Determine which of the following sequences converge and which diverge. If a sequence converges, indicate its limit.
   (a) $\{b_n\}$ (b) $\{3a_n - 1\}$ (c) $\{b_n^2\}$
   (d) $\{a_n + b_n\}$ (e) $\left\{\dfrac{1}{a_n^2 + 3}\right\}$ (f) $\left\{\dfrac{b_n}{1000}\right\}$
4. Suppose that $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ are sequences such that $a_n \le b_n \le c_n$ for all $n \ge 10$, and that $\{a_n\}$ and $\{c_n\}$ both converge to 12. Then the ______ Theorem for Sequences implies that $\{b_n\}$ converges to ______.

EXERCISE SET 9.1

Graphing Utility

1. In each part, find a formula for the general term of the sequence, starting with $n = 1$.
   (a) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ (b) $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \ldots$
   (c) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$ (d) $\frac{1}{\sqrt{\pi}}, \frac{4}{\sqrt[3]{\pi}}, \frac{9}{\sqrt[4]{\pi}}, \frac{16}{\sqrt[5]{\pi}}, \ldots$
2. In each part, find two formulas for the general term of the sequence, one starting with $n = 1$ and the other with $n = 0$.
   (a) $1, -r, r^2, -r^3, \ldots$ (b) $r, -r^2, r^3, -r^4, \ldots$
3. (a) Write out the first four terms of the sequence $\{1 + (-1)^n\}$, starting with $n = 0$.
   (b) Write out the first four terms of the sequence $\{\cos n\pi\}$, starting with $n = 0$.
   (c) Use the results in parts (a) and (b) to express the general term of the sequence $4, 0, 4, 0, \ldots$ in two different ways, starting with $n = 0$.
4. In each part, find a formula for the general term using factorials and starting with $n = 1$.
   (a) $1 \cdot 2, 1 \cdot 2 \cdot 3 \cdot 4, 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6, 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8, \ldots$
   (b) $1, 1 \cdot 2 \cdot 3, 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5, 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7, \ldots$

5–6 Let f be the function $f(x) = \cos\left(\frac{\pi}{2}x\right)$ and define sequences $\{a_n\}$ and $\{b_n\}$ by $a_n = f(2n)$ and $b_n = f(2n+1)$.

5. (a) Does $\lim_{x \to +\infty} f(x)$ exist? Explain.
   (b) Evaluate a_1, a_2, a_3, a_4, and a_5.
   (c) Does $\{a_n\}$ converge? If so, find its limit.
6. (a) Evaluate b_1, b_2, b_3, b_4, and b_5.
   (b) Does $\{b_n\}$ converge? If so, find its limit.
   (c) Does $\{f(n)\}$ converge? If so, find its limit.

7–22 Write out the first five terms of the sequence, determine whether the sequence converges, and if so find its limit.

7. $\left\{\dfrac{n}{n+2}\right\}_{n=1}^{+\infty}$ 8. $\left\{\dfrac{n^2}{2n+1}\right\}_{n=1}^{+\infty}$ 9. $\{2\}_{n=1}^{+\infty}$

10. $\left\{\ln\left(\dfrac{1}{n}\right)\right\}_{n=1}^{+\infty}$ 11. $\left\{\dfrac{\ln n}{n}\right\}_{n=1}^{+\infty}$ 12. $\left\{n \sin\dfrac{\pi}{n}\right\}_{n=1}^{+\infty}$

13. $\{1 + (-1)^n\}_{n=1}^{+\infty}$ 14. $\left\{\dfrac{(-1)^{n+1}}{n^2}\right\}_{n=1}^{+\infty}$

15. $\left\{(-1)^n \dfrac{2n^3}{n^3+1}\right\}_{n=1}^{+\infty}$ 16. $\left\{\dfrac{n}{2^n}\right\}_{n=1}^{+\infty}$

17. $\left\{\dfrac{(n+1)(n+2)}{2n^2}\right\}_{n=1}^{+\infty}$ 18. $\left\{\dfrac{\pi^n}{4^n}\right\}_{n=1}^{+\infty}$

19. $\{n^2 e^{-n}\}_{n=1}^{+\infty}$ 20. $\{\sqrt{n^2+3n} - n\}_{n=1}^{+\infty}$

21. $\left\{\left(\dfrac{n+3}{n+1}\right)^n\right\}_{n=1}^{+\infty}$ 22. $\left\{\left(1 - \dfrac{2}{n}\right)^n\right\}_{n=1}^{+\infty}$

23–30 Find the general term of the sequence, starting with $n = 1$, determine whether the sequence converges, and if so find its limit.

23. $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots$ 24. $0, \frac{1}{2^2}, \frac{2}{3^2}, \frac{3}{4^2}, \ldots$

25. $\frac{1}{3}, -\frac{1}{9}, \frac{1}{27}, -\frac{1}{81}, \ldots$ 26. $-1, 2, -3, 4, -5, \ldots$

27. $\left(1 - \frac{1}{2}\right), \left(\frac{1}{3} - \frac{1}{2}\right), \left(\frac{1}{3} - \frac{1}{4}\right), \left(\frac{1}{5} - \frac{1}{4}\right), \ldots$

28. $3, \frac{3}{2}, \frac{3}{2^2}, \frac{3}{2^3}, \ldots$

29. $(\sqrt{2} - \sqrt{3}), (\sqrt{3} - \sqrt{4}), (\sqrt{4} - \sqrt{5}), \ldots$

30. $\frac{1}{3^5}, -\frac{1}{3^6}, \frac{1}{3^7}, -\frac{1}{3^8}, \ldots$