

QUICK CHECK EXERCISES 9.3 (See page 539 for answers.)

1. In mathematics, the terms “sequence” and “series” have different meanings: a _____ is a succession, whereas a _____ is a sum.

2. Consider the series

$$\sum_{k=1}^{\infty} \frac{1}{2^k}$$

If $\{s_n\}$ is the sequence of partial sums for this series, then

$s_1 = \underline{\hspace{2cm}}, s_2 = \underline{\hspace{2cm}}, s_3 = \underline{\hspace{2cm}},$
 $s_4 = \underline{\hspace{2cm}},$ and $s_n = \underline{\hspace{2cm}}.$

3. What does it mean to say that a series $\sum u_k$ converges?

4. A geometric series is a series of the form

$$\sum_{k=0}^{\infty} \underline{\hspace{2cm}}$$

This series converges to _____ if _____. This series diverges if _____.

5. The harmonic series has the form

$$\sum_{k=1}^{\infty} \underline{\hspace{2cm}}$$

Does the harmonic series converge or diverge?

EXERCISE SET 9.3  CAS

1–2 In each part, find exact values for the first four partial sums, find a closed form for the n th partial sum, and determine whether the series converges by calculating the limit of the n th partial sum. If the series converges, then state its sum. ■

1. (a) $2 + \frac{2}{5} + \frac{2}{5^2} + \dots + \frac{2}{5^{k-1}} + \dots$

(b) $\frac{1}{4} + \frac{2}{4} + \frac{2^2}{4} + \dots + \frac{2^{k-1}}{4} + \dots$

(c) $\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{(k+1)(k+2)} + \dots$

2. (a) $\sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^k$ (b) $\sum_{k=1}^{\infty} 4^{k-1}$ (c) $\sum_{k=1}^{\infty} \left(\frac{1}{k+3} - \frac{1}{k+4}\right)$

3–14 Determine whether the series converges, and if so find its sum. ■

3. $\sum_{k=1}^{\infty} \left(-\frac{3}{4}\right)^{k-1}$

4. $\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^{k+2}$

5. $\sum_{k=1}^{\infty} (-1)^{k-1} \frac{7}{6^{k-1}}$

6. $\sum_{k=1}^{\infty} \left(-\frac{3}{2}\right)^{k+1}$

7. $\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}$

8. $\sum_{k=1}^{\infty} \left(\frac{1}{2^k} - \frac{1}{2^{k+1}}\right)$

9. $\sum_{k=1}^{\infty} \frac{1}{9k^2 + 3k - 2}$

10. $\sum_{k=2}^{\infty} \frac{1}{k^2 - 1}$

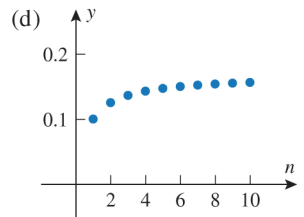
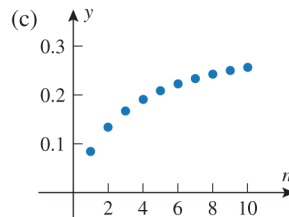
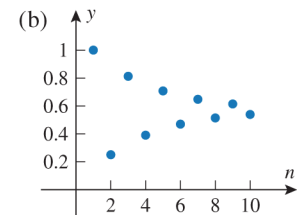
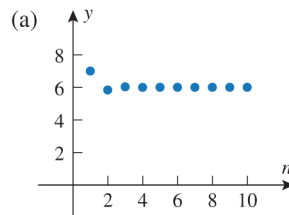
11. $\sum_{k=3}^{\infty} \frac{1}{k-2}$

12. $\sum_{k=5}^{\infty} \left(\frac{e}{\pi}\right)^{k-1}$

13. $\sum_{k=1}^{\infty} \frac{4^{k+2}}{7^{k-1}}$

14. $\sum_{k=1}^{\infty} 5^{3k} 7^{1-k}$

15. Match a series from one of Exercises 3, 5, 7, or 9 with the graph of its sequence of partial sums.



16. Match a series from one of Exercises 4, 6, 8, or 10 with the graph of its sequence of partial sums.

