EXERCISE SET 9.4 Graphing Utility C CAS

1. Use Theorem 9.4.3 to find the sum of each series.
(a) $\left(\frac{1}{2}+\frac{1}{4}\right)+\left(\frac{1}{2^2}+\frac{1}{4^2}\right)+\cdots+\left(\frac{1}{2^k}+\frac{1}{4^k}\right)+\cdots$
(b) $\sum_{k=1}^{\infty}\left(\frac{1}{5^k}-\frac{1}{k(k+1)}\right)$

2. Use Theorem 9.4.3 to find the sum of each series.
(a) $\sum_{k=2}^{\infty}\left[\frac{1}{k^2-1}-\frac{7}{10^{k-1}}\right]$ (b) $\sum_{k=1}^{\infty}\left[7^{-k}3^{k+1}-\frac{2^{k+1}}{5^k}\right]$

3–4 For each given p-series, identify p and determine whether the series converges.

3. (a) $\sum_{k=1}^{\infty}\frac{1}{k^3}$ (b) $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}$ (c) $\sum_{k=1}^{\infty}k^{-1}$ (d) $\sum_{k=1}^{\infty}k^{-2/3}$

4. (a) $\sum_{k=1}^{\infty}k^{-4/3}$ (b) $\sum_{k=1}^{\infty}\frac{1}{\sqrt[4]{k}}$ (c) $\sum_{k=1}^{\infty}\frac{1}{\sqrt[3]{k^5}}$ (d) $\sum_{k=1}^{\infty}\frac{1}{k^{\pi}}$

5–6 Apply the divergence test and state what it tells you about the series.

5. (a) $\sum_{k=1}^{\infty}\frac{k^2+k+3}{2k^2+1}$ (b) $\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^k$
(c) $\sum_{k=1}^{\infty}\cos k\pi$ (d) $\sum_{k=1}^{\infty}\frac{1}{k!}$

6. (a) $\sum_{k=1}^{\infty}\frac{k}{e^k}$ (b) $\sum_{k=1}^{\infty}\ln k$
(c) $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}$ (d) $\sum_{k=1}^{\infty}\frac{\sqrt{k}}{\sqrt{k}+3}$

7–8 Confirm that the integral test is applicable and use it to determine whether the series converges.

7. (a) $\sum_{k=1}^{\infty}\frac{1}{5k+2}$ (b) $\sum_{k=1}^{\infty}\frac{1}{1+9k^2}$

8. (a) $\sum_{k=1}^{\infty}\frac{k}{1+k^2}$ (b) $\sum_{k=1}^{\infty}\frac{1}{(4+2k)^{3/2}}$

9–24 Determine whether the series converges.

9. $\sum_{k=1}^{\infty}\frac{1}{k+6}$ 10. $\sum_{k=1}^{\infty}\frac{3}{5k}$ 11. $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k+5}}$

12. $\sum_{k=1}^{\infty}\frac{1}{\sqrt[k]{e}}$ 13. $\sum_{k=1}^{\infty}\frac{1}{\sqrt[3]{2k-1}}$ 14. $\sum_{k=3}^{\infty}\frac{\ln k}{k}$

15. $\sum_{k=1}^{\infty}\frac{k}{\ln(k+1)}$ 16. $\sum_{k=1}^{\infty}ke^{-k^2}$ 17. $\sum_{k=1}^{\infty}\left(1+\frac{1}{k}\right)^{-k}$

18. $\sum_{k=1}^{\infty}\frac{k^2+1}{k^2+3}$ 19. $\sum_{k=1}^{\infty}\frac{\tan^{-1}k}{1+k^2}$ 20. $\sum_{k=1}^{\infty}\frac{1}{\sqrt{k^2+1}}$

21. $\sum_{k=1}^{\infty}k^2\sin^2\left(\frac{1}{k}\right)$ 22. $\sum_{k=1}^{\infty}k^2e^{-k^3}$

23. $\sum_{k=5}^{\infty}7k^{-1.01}$ 24. $\sum_{k=1}^{\infty}\operatorname{sech}^2 k$

25–26 Use the integral test to investigate the relationship between the value of p and the convergence of the series.

25. $\sum_{k=2}^{\infty}\frac{1}{k(\ln k)^p}$ 26. $\sum_{k=3}^{\infty}\frac{1}{k(\ln k)[\ln(\ln k)]^p}$

FOCUS ON CONCEPTS

27. Suppose that the series $\sum u_k$ converges and the series $\sum v_k$ diverges. Show that the series $\sum(u_k+v_k)$ and $\sum(u_k-v_k)$ both diverge. [*Hint:* Assume that $\sum(u_k+v_k)$ converges and use Theorem 9.4.3 to obtain a contradiction.]

28. Find examples to show that if the series $\sum u_k$ and $\sum v_k$ both diverge, then the series $\sum(u_k+v_k)$ and $\sum(u_k-v_k)$ may either converge or diverge.

29–30 Use the results of Exercises 27 and 28, if needed, to determine whether each series converges or diverges.

29. (a) $\sum_{k=1}^{\infty}\left[\left(\frac{2}{3}\right)^{k-1}+\frac{1}{k}\right]$ (b) $\sum_{k=1}^{\infty}\left[\frac{1}{3k+2}-\frac{1}{k^{3/2}}\right]$

30. (a) $\sum_{k=2}^{\infty}\left[\frac{1}{k(\ln k)^2}-\frac{1}{k^2}\right]$ (b) $\sum_{k=2}^{\infty}\left[ke^{-k^2}+\frac{1}{k\ln k}\right]$

31–34 True–False Determine whether the statement is true or false. Explain your answer.

31. If $\sum u_k$ converges to L, then $\sum(1/u_k)$ converges to $1/L$.

32. If $\sum cu_k$ diverges for some constant c, then $\sum u_k$ must diverge.

33. The integral test can be used to prove that a series diverges.

34. The series $\sum_{k=1}^{\infty}\frac{1}{p^k}$ is a p-series.

C 35. Use a CAS to confirm that

$$\sum_{k=1}^{\infty}\frac{1}{k^2}=\frac{\pi^2}{6} \quad\text{and}\quad \sum_{k=1}^{\infty}\frac{1}{k^4}=\frac{\pi^4}{90}$$

and then use these results in each part to find the sum of the series.

(a) $\sum_{k=1}^{\infty}\frac{3k^2-1}{k^4}$ (b) $\sum_{k=3}^{\infty}\frac{1}{k^2}$ (c) $\sum_{k=2}^{\infty}\frac{1}{(k-1)^4}$