5–10 Use the limit comparison test to determine whether the series converges.

5. $\sum_{k=1}^{\infty} \frac{4k^2 - 2k + 6}{8k^7 + k - 8}$ **6.** $\sum_{k=1}^{\infty} \frac{1}{9k + 6}$

7. $\sum_{k=1}^{\infty} \frac{5}{3^k + 1}$ **8.** $\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$

9. $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{8k^2 - 3k}}$ **10.** $\sum_{k=1}^{\infty} \frac{1}{(2k+3)^{17}}$

11–16 Use the ratio test to determine whether the series converges. If the test is inconclusive, then say so.

11. $\sum_{k=1}^{\infty} \frac{3^k}{k!}$ **12.** $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$ **13.** $\sum_{k=1}^{\infty} \frac{1}{5k}$

14. $\sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k$ **15.** $\sum_{k=1}^{\infty} \frac{k!}{k^3}$ **16.** $\sum_{k=1}^{\infty} \frac{k}{k^2 + 1}$

17–20 Use the root test to determine whether the series converges. If the test is inconclusive, then say so.

17. $\sum_{k=1}^{\infty} \left(\frac{3k+2}{2k-1}\right)^k$ **18.** $\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$

19. $\sum_{k=1}^{\infty} \frac{k}{5^k}$ **20.** $\sum_{k=1}^{\infty} (1 - e^{-k})^k$

21–24 True–False Determine whether the statement is true or false. Explain your answer.

21. The limit comparison test decides convergence based on a limit of the quotient of consecutive terms in a series.

22. If $\lim_{k\to+\infty} (u_{k+1}/u_k) = 5$, then $\sum u_k$ diverges.

23. If $\lim_{k\to+\infty} (k^2 u_k) = 5$, then $\sum u_k$ converges.

24. The root test decides convergence based on a limit of kth roots of terms in the sequence of partial sums for a series.

25–49 Use any method to determine whether the series converges.

25. $\sum_{k=0}^{\infty} \frac{7^k}{k!}$ **26.** $\sum_{k=1}^{\infty} \frac{1}{2k+1}$ **27.** $\sum_{k=1}^{\infty} \frac{k^2}{5^k}$

28. $\sum_{k=1}^{\infty} \frac{k!\,10^k}{3^k}$ **29.** $\sum_{k=1}^{\infty} k^{50} e^{-k}$ **30.** $\sum_{k=1}^{\infty} \frac{k^2}{k^3 + 1}$

31. $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^3 + 1}$ **32.** $\sum_{k=1}^{\infty} \frac{4}{2 + 3^k k}$

33. $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k(k+1)}}$ **34.** $\sum_{k=1}^{\infty} \frac{2 + (-1)^k}{5^k}$

35. $\sum_{k=1}^{\infty} \frac{2 + \sqrt{k}}{(k+1)^3 - 1}$ **36.** $\sum_{k=1}^{\infty} \frac{4 + |\cos x|}{k^3}$

37. $\sum_{k=1}^{\infty} \frac{1}{1 + \sqrt{k}}$ **38.** $\sum_{k=1}^{\infty} \frac{k!}{k^k}$ **39.** $\sum_{k=1}^{\infty} \frac{\ln k}{e^k}$

40. $\sum_{k=1}^{\infty} \frac{k!}{e^{k^2}}$ **41.** $\sum_{k=0}^{\infty} \frac{(k+4)!}{4!\,k!\,4^k}$ **42.** $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$

43. $\sum_{k=1}^{\infty} \frac{1}{4 + 2^{-k}}$ **44.** $\sum_{k=1}^{\infty} \frac{\sqrt{k}\ln k}{k^3 + 1}$ **45.** $\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{k^2}$

46. $\sum_{k=1}^{\infty} \frac{5^k + k}{k! + 3}$ **47.** $\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}$ **48.** $\sum_{k=1}^{\infty} \frac{[\pi(k+1)]^k}{k^{k+1}}$

49. $\sum_{k=1}^{\infty} \frac{\ln k}{3^k}$

50. For what positive values of α does the series $\sum_{k=1}^{\infty} (\alpha^k / k^\alpha)$ converge?

51–52 Find the general term of the series and use the ratio test to show that the series converges.

51. $1 + \frac{1 \cdot 2}{1 \cdot 3} + \frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 3 \cdot 5 \cdot 7} + \cdots$

52. $1 + \frac{1 \cdot 3}{3!} + \frac{1 \cdot 3 \cdot 5}{5!} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{7!} + \cdots$

53. Show that $\ln x < \sqrt{x}$ if $x > 0$, and use this result to investigate the convergence of

(a) $\sum_{k=1}^{\infty} \frac{\ln k}{k^2}$ (b) $\sum_{k=2}^{\infty} \frac{1}{(\ln k)^2}$

FOCUS ON CONCEPTS

54. (a) Make a conjecture about the convergence of the series $\sum_{k=1}^{\infty} \sin(\pi/k)$ by considering the local linear approximation of $\sin x$ at $x = 0$.
(b) Try to confirm your conjecture using the limit comparison test.

55. (a) We will see later that the polynomial $1 - x^2/2$ is the "local quadratic" approximation for $\cos x$ at $x = 0$. Make a conjecture about the convergence of the series

$$\sum_{k=1}^{\infty} \left[1 - \cos\left(\frac{1}{k}\right)\right]$$

by considering this approximation.
(b) Try to confirm your conjecture using the limit comparison test.

56. Let $\sum a_k$ and $\sum b_k$ be series with positive terms. Prove:
(a) If $\lim_{k\to+\infty} (a_k/b_k) = 0$ and $\sum b_k$ converges, then $\sum a_k$ converges.
(b) If $\lim_{k\to+\infty} (a_k/b_k) = +\infty$ and $\sum b_k$ diverges, then $\sum a_k$ diverges.