Makeup Exam Answers

1. \( \mathbf{u} \times \mathbf{w} = (2, 1, 0) \times (4, 0, 1) = (1, -2, -4)\)
   \[ V = |\mathbf{u} \times \mathbf{w} \cdot \mathbf{v}| = (1, -2, -4) \cdot (1, -3, 1) = 3 \]

2. \( \langle 1, 2, -1 \rangle \) is \( \| \) to \( L_1, L_2 \)
   \( \langle 3, 4, 0 \rangle - \langle -2, 3, 4 \rangle = \langle 5, 1, -4 \rangle \) is \( \| \) to the plane, not to \( \langle 1, 2, -1 \rangle \)
   Hence \( 7(x - 3) + (y - 4) + 9z = 0 \) is the equation of the plane

3. \( \mathbf{r}(0) = \langle 1, 1, 0 \rangle \)
   \( \mathbf{r}'(0) = (-2e^{-2t}, -sin \, t, 3 \cos \, t) \bigg|_{t=0} = \langle -2, 0, 3 \rangle \)
   Tangent line: \( \mathbf{t}(t) = \langle 1, 1, 0 \rangle + t\langle -2, 0, 3 \rangle \)

4. \( (2, 2, 2) \) is the point of intersection
   Between line and normal: angle = \( \arccos \frac{(1,1,1) \cdot (3,-2,1)}{\sqrt{1^2 + 1^2 + 1^2}} = \arccos \frac{1}{\sqrt{10}} \)
   Between line and plane: angle = \( \frac{\pi}{2} - \arccos \frac{1}{\sqrt{10}} \)

5. \( f_{xx} = 4y^2, f_{yy} = 12x^2y + 2, f_{xy} = f_{yx} = 12xy^2 \)

6. \( \sin(xy) \approx \frac{\sqrt{3}}{2} + \frac{\pi}{2}(x - \frac{1}{3}) + \frac{1}{6}(y - \pi), \quad (x, y) \) near \( \left( \frac{1}{3}, \pi \right) \)

7. \( y = \frac{1}{2} x^2 \) is the level curve through \( (-2, 2) \), \( \nabla f(-2,2) = \langle \frac{1}{2}, \frac{1}{4} \rangle \)

8. For positive \( x, y, z \), with \( xyz = 32 \), one needs to minimize \( xy + 2yz + 2xz \). In other words, we seek a point of absolute minimum of \( A(x, y) = xy + \frac{64}{x} + 64y \), where \( x, y > 0 \).
   Since \( A_x = y - \frac{64}{x^2} \) and \( A_y = x - \frac{64}{y^2} \), the only critical point of \( A \) is \( x = y = 4 \). The Second Partials Test confirms that \( A \) has a local minimum at \( (4, 4) \). An argument is needed to verify that \( (4, 4) \) is, in fact, a point of absolute minimum (see Problem B, Quiz 7), so that \( 4 \times 4 \times 2 \) are optimum dimensions.