Quiz 3 Answers

A) Examine components: \( x = \cos(\pi t) \) is defined for all \( t \), \( y = -\ln t \) is defined for \( t > 0 \), and \( z = \sqrt{2 - t} \) is defined for \( t \leq 2 \). The domain of \( \vec{r}'(t) \) is therefore \( 0 < t \leq 2 \).

B) The graph of \( \vec{r}'(t) \) is a helix. Starting at \((2, 0, 0)\) it winds around the surface of the elliptic cylinder \((\frac{x}{2})^2 + (\frac{y}{3})^2 = 1\). At \( t \) increases, \( \vec{r}'(t) \) gains elevation rotating clockwise.

C) Observe that \((t \sin t)^2 + (t \cos t)^2 = t^2(\sin^2 t + \cos^2 t) = t^2\). Hence, for every \( t \), the point \((t \sin t, t \cos t, t^2)\) lies on the surface of the elliptic paraboloid \( x^2 + y^2 = z \).

D) The tangent line passes through the point \( r(\frac{1}{3}) = (1, \sqrt{3}, 1) \) and is parallel to the instantaneous velocity vector \( \vec{r}'(\frac{1}{3}) = (-\pi \sqrt{3}, \pi, 3) \).

Its vector equation is
\[
\vec{r}_{\text{tan}}(s) = (1, \sqrt{3}, 1) + s(-\pi \sqrt{3}, \pi, 3), \quad -\infty < s < \infty,
\]
and its parametric equations are:
\[
x = 1 - \pi \sqrt{3}s, \quad y = \sqrt{3} + \pi s, \quad z = 1 + 3s.
\]

E)
\[
\int (1, 0, t) dt = (t + c_1, c_2, \frac{1}{2} t^2 + c_3) = (t, 0, \frac{1}{2} t^2) + \vec{c}
\]
\[
\int_0^1 (1, 0, t) dt = (1, 0, \frac{1}{2})
\]

F) Letting \( x = t, y = t^2, z = -3t \), we infer that the point \( r(t) \) lies in the plane \( 2x - y + z + 2 = 0 \) if and only if \( 2t - t^2 - 3t + 2 = 0 \). Thus \( t^2 + t - 2 = 0 \) and so \( t = 1, -2 \). This gives two points of intersection: \((1, 1, -3)\) and \((-2, 4, 6)\). For each point of intersection we need to determine the acute angle between the normal \( \vec{n} = (2, -1, 1) \) and tangent \( \vec{r}''(t) = (1, 2t, -3) \) directions. We have:
\[
\theta(1) = \arccos \frac{|2+1+(-1)-2+1|}{\sqrt{6}\sqrt{14}} = \arccos \frac{1}{2} \frac{\sqrt{3}}{\sqrt{7}}
\]
and
\[
\theta(-2) = \arccos \frac{|2+1+(-1)-(-4)+1-(-3)|}{\sqrt{6}\sqrt{26}} = \arccos \frac{1}{2} \frac{\sqrt{3}}{\sqrt{13}}.
\]
Approximations are \( \theta(1) \approx 71^\circ, \theta(-2) \approx 76^\circ \).