A) In symbols, “Y follows from X” is “X ⇒ Y”. The contrapositive of the inverse of “X ⇒ Y” is “Y ⇒ X”, the converse statement.

B) “(X AND ¬Y) ⇒ Z” is false only if “X AND ¬Y” is true and Z is false, i.e. if X is true, Y is false, and Z is false.

“X ⇒ (Y OR Z)” is false only if X is true and “Y OR Z” is false, i.e. if X is true, Y is false, and Z is false. Hence the two statements have identical truth tables and so are equivalent.

C) “∀ four real numbers ∃ two that are equal mod 3.” In words: “Among any four real numbers there exist two that are equal modulo 3.”

(The statement is false: 0, 1, 2, and π are all different mod 3.
But it is true for integers.)

HOMEWORK 2

**Proposition.** If n − 1, n, and n + 1 are three consecutive integers, then 9 divides the sum of their cubes.

**Proof of Proposition.** Let σ be the sum of cubes. A short calculation gives

\[ \sigma = (n - 1)^3 + n^3 + (n + 1)^3 \]
\[ = n^3 - 3n^2 + 3n - 1 + n^3 + 3n^2 + 3n + 1 \]
\[ = 3n(n^2 + 2). \]

If n is a multiple of 3, then σ is a multiple of 9. Otherwise, \( n^2 + 2 \) is a multiple of 3 and so \( \sigma \) is again a multiple of 9. Hence 9 divides \( \sigma \) in all cases. □