**EXAM II ANSWERS**

1 A) True.
1 B) False. (proof by cases)
1 C) True. (proof by elimination)
1 D) True.
1 E) True.
1 F) False.
1 G) True.
1 H) True.
1 I) True.
1 J) False.

2 A) \( \text{GCD}(522, 48) = 6 \).
2 B) \(-1 \times 522 + 11 \times 48 = 6\).
2 C) Assume that there exist two integers \( m \) and \( n \) such that \( 522m + 48n = 5 \). Then, since the left side is a multiple of 6, the right side is also a multiple of 6. But 6 does not divide 5, a contradiction. Therefore no such \( m \) and \( n \) exist.

3 First of all, such an integer exists: \( n = 1000 \). To prove uniqueness, note that, if \( n \) divides 1000 then \( n \leq 1000 \), and if 1000 divides \( n > 0 \) then \( 1000 \leq n \). Hence \( n = 1000 \) is the only option.

4. The formula holds for \( n = 1 \),
\[
1 \cdot 1! = (1 + 1)! - 1.
\]
Suppose that the formula holds for \( n = k \). Then, for \( n = k + 1 \), we have by the induction hypothesis
\[
1! + \cdots + n \cdot n! + (n+1) \cdot (n+1)! = (n+1)! - 1 + (n+1) \cdot (n+1)! = (n+1)![(n+2) - 1] = (n+2)! - 1.
\]
Hence, by the method of mathematical induction, the formula holds for all \( n \).