HOMEWORK 5, DUE MAY 18.

1. Without using a computing device, try to determine the number of real solutions of the equation \(x^4 - x - 1 = 0\). What are their approximate locations (suggest upper and lower bounds)?

2. Let a sequence of approximations \(x_0, x_1, x_2, \ldots\) approaching \(\alpha\) be obtained using bisection. Is it necessarily true that \(|x_{n+1} - \alpha| \leq \frac{1}{2}|x_n - \alpha|\)? Is it true that \(|x_{n+1} - \alpha| \leq \frac{1}{2}\max\{|x_n - \alpha|, |x_{n-1} - \alpha|\}\)?

3. Use Newton’s method to approximate the negative root of the equation \(x^4 - 3x^2 + 75x - 10000 = 0\) to 5 correct digits.

4. For the cubic polynomial \(x^3 - x\), which initial values \(x_0\) of the Newton iteration correspond to which roots?