1. Let \( p_\varepsilon(x) = x^3 - x + \varepsilon \) and let \( \alpha(\varepsilon) \) be its largest root.

(a) Find \( \alpha(0) \) and \( \alpha'(0) \). (b) Estimate \( \alpha(\varepsilon) \) for small \( \varepsilon \).

(c) Suggest an iteration (and an initial guess) converging to \( \alpha(\varepsilon) \) for small \( \varepsilon \).

2. Given is \( F(x) = \frac{1}{2} \left( x + \frac{2.89}{x} \right) \).

(a) Determine the fixed points of \( F \).

(b) Argue that \( 1 < F(x) < 2 \) for \( 1 \leq x \leq 2 \).

(c) Find the maximum of \( |F'(x)| \) on \([1, 2]\).

(d) Given \( 1 \leq x_0 \leq 2 \), what is the behavior of the iteration \( x_{n+1} = F(x_n) \)?

(e) Can you find the order and rate of convergence?