EXERCISES, week 2.

1. Find the quotient and the remainder from dividing $x^2 + 2x + 1$ by $x - 2$.

2. Evaluate $x^2 + x^4$ at $x = x_0$ as efficiently as possible.

3. Let $p(x) = a_0 + a_1 x + \cdots + a_n x^n$. Evaluation of $p(x)$ at $x = \omega$ with nested multiplication yields the numbers
   
   $b_n = a_n, \ b_{n-1} = a_{n-1} + \omega b_n, \ \ldots, \ b_1 = a_1 + \omega b_2, \ b_0 = a_0 + \omega b_1.$

   Let $q(x) = b_1 + b_2 x \cdots + b_n x^{n-1}$. Verify that $b_0 = p(\omega)$ and that
   
   $p(x) = p(\omega) + (x - \omega)q(x), \ \text{so that} \ \ q(x)$ is the quotient and $p(\omega)$ is the remainder
   
   from dividing $p(x)$ by $(x - \omega)$.

4. Convert $(123)_{10}$ to binary form and $(100100)_2$ to decimal form.

5. Convert $(12.5)_{10}$ to binary form and $(1.101)_2$ to decimal form.

6. Find the binary representation of $1/10$ and of $\sqrt{2}$ (at least a few binary digits).

7. Convert the hexadecimal number $(AC.17)_{16}$ to binary form.

8. Convert $(1010.1001)_2$ to hexadecimal form.

9. Find the decimal representation of $1/7$.

10. (Extra credit) Choose an irrational number $\alpha > 0$. Use a graphing utility to plot the lattice of points where the family of horizontal lines $y = q$ $(q = 1, 2, 3, \ldots)$ meets the family of parallel lines $x = \alpha y - p$ $(p = 0, \pm1, \pm2, \ldots)$. Verify that, for different choices of a natural $N$, the rectangle with vertices $(-\frac{1}{N}, 0), (\frac{1}{N}, 0), \ (-\frac{1}{N}, N + 1), \ (\frac{1}{N}, N + 1)$ always happens to contain at least one lattice point in its interior. Do you see an explanation for this? (Hint: Pigeonhole principle.)