EXERCISES, week 5.

1. Find the 5-th Taylor polynomial of \( f(x) = (1 + x)^{-1}, \) \( x_0 = 0. \)

2. Does \( f(x) = x^{1/3} \) have the linear Taylor approximation at \( x_0 = 0? \ x_0 = 1? \)

3. For \( f(x) = e^x, \) find a cubic polynomial \( q(x) \) with \( q(0) = f(0), \) \( q'(0) = f'(0), \)
\( q(1) = f(1), \) and \( q'(1) = f'(1). \) Write the third Taylor polynomial \( T_3(x) \) of \( e^x \)
centered at the origin. Use a graphing utility to compare \( f(x), \) \( q(x), \) and \( T_3(x) \)
on the interval \([0, 1].\)

4. Bound the error in the approximation \( \sin x \approx x \) for \( 0 \leq x \leq \frac{\pi}{4}. \)

5. Use the Taylor approximation to find \( \lim_{x \to 0} \frac{e^x - 1}{x}. \)

6. How large should \( n \) be to have \( e^x - \sum_{k=0}^{n} \frac{x^k}{k!} < 10^{-2} \) for \( 0 \leq x \leq 1? \)

HOMEWORK 4, due May 13.

A) Find the linear and quadratic Taylor polynomials for \( f(x) = \sqrt{x}, \) \( x_0 = 1. \)
Check that \( f(x) - T_1(x) \) vanishes to order 1 and \( f(x) - T_2(x) \) vanishes to order 2 at \( x_0. \)

B) Use Taylor’s approximation to evaluate \( \int_0^1 \frac{e^x - 1}{x} \) \( dx \) within an accuracy of \( 10^{-6}.\)