EXERCISES, WEEK 6.

1. Use the bisection method to find the root of \( x = e^{-x} \) with an accuracy of 0.0001. How many iterations did you need?

2. Use Newton’s method to find the root of \( x = e^{-x} \) with an accuracy of 0.0001. How many iterations did you need?

3. Consider applying Newton’s method to find the root \( \alpha = 0 \) of \( \sin x = 0 \). Find the order and the rate of convergence. Can you determine the set of convergence (the set of all \( x_0 \) such that \( x_n \to 0 \))?

4. (Extra credit) Consider applying Newton’s method to the equation \( x^2 + 1 = 0 \). Analyze possible outcomes for different choices of \( x_0 \). Use a computer for hints and illustration.

HOMEWORK 5, DUE MAY 20.

The computation of \( \sqrt{a} \), \( a > 0 \), is often based on Newton’s method.

A) Set up the Newton iteration for solving \( x^2 - a = 0 \), and show that it can be written in the form \( x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n}) \), \( n \geq 0 \).

B) Derive the formula \( \sqrt{a} - x_{n+1} = -\frac{1}{2x_n}(\sqrt{a} - x_n)^2 \).

C) Derive the formula \( \text{Rel}(x_{n+1}) = -\frac{\sqrt{a}}{2x_n^2}\text{Rel}(x_n)^2 \).

D) How does the behavior of \( x_n \) depend on the initial guess \( x_0 \)?

E) Determine the rate and order of convergence.

F) If \( a = 0 \), how is your analysis different? Supply details.