

HOMEWORK 1

due Wednesday, April 10

1. Let a, b, c be real numbers with $a \neq 0$ and $b^2 < 4ac$. Show that the two roots of the equation $ax^2 + bx + c = 0$ are complex conjugates of each other.
2. Consider the cubic equation $x^3 = 15x + 4$. Show that $x = \sqrt[3]{u} + \sqrt[3]{v}$ is a solution to the equation if $u + v = 4$ and $uv = 125$. Determine u, v and x .
3. Let z_1, z_2, z_3, z_4 be points in the complex plane and let $z_1 - z_2 + z_3 - z_4 = 0$. Describe the geometric configuration determined by the points.
4. Find and plot all cube roots of i and eighth roots of 1 .
5. Let z_1, z_2, z_3 be distinct points on the unit circle $|z| = 1$. Prove that

$$\arg \frac{z_1}{z_2} = 2 \arg \frac{z_3 - z_1}{z_3 - z_2}.$$