Complex Analysis, Spring 2019

Grinshpan

Homework 2

due Wednesday, April 17

1. At which points is $f(z) = \overline{z}^2$ differentiable?

2. Prove that the Cauchy–Riemann equations in polar coordinates are

$$r\,\frac{\partial u}{\partial r} = \frac{\partial v}{\partial \theta}, \quad \frac{\partial u}{\partial \theta} = -r\,\frac{\partial v}{\partial r}.$$

3. Let $z_1 = 1$ and $z_2 = i$. If $f(z) = z^2$, can you find z_0 in $[z_1,\ z_2]$ such that

$$f'(z_0) = \frac{f(z_1) - f(z_2)}{z_1 - z_2}\ ?$$

What if $f(z) = z^3$?

4. For which values of the real constants a, b, c, d is the function

$$u(x, y) = ax^3 + bx^2y + cxy^2 + dy^3$$

harmonic? Determine a harmonic conjugate of u in the cases where it is harmonic.

5. Let $f(z) = u + iv$ be a holomorphic function with nonvanishing derivative.
Prove that the level curves $\operatorname{Re} f(x + iy) = u_0$ and $\operatorname{Im} f(x + iy) = v_0$ are orthogonal.