Complex Analysis, Spring 2019

Grinshpan

Homework 3

due Wednesday, April 24

1. Find the image of the half-plane $\operatorname{Re} z > 0$ under the linear-fractional transformation that maps $0,\ i,\ -i$ to $1,\ -1,\ 0,$ respectively.

2. Prove that a linear-fractional transformation maps the half-plane $\operatorname{Im} > 0$ onto itself if and only if it is induced by a matrix with real entries and unit determinant.

3. Given four distinct points $z_1,\ z_2,\ z_3,\ z_4$ in the extended complex plane, their cross ratio, which is denoted by $(z_1,\ z_2;\ z_3,\ z_4)$, is defined to be the image of z_4 under the linear-fractional transformation that sends $z_1,\ z_2,\ z_3$ to $\infty,\ 0,\ 1,$ respectively. Prove that if φ is a linear-fractional transformation, then

$$(z_1,\ z_2;\ z_3,\ z_4) = (\varphi(z_1),\ \varphi(z_2);\ \varphi(z_3),\ \varphi(z_4)).$$

4. Find all roots of the equation $\cos z = 2$.

5. Describe the images of the lines $x = x_0$ and $y = y_0$ under the mapping $z \mapsto \cos z$.