

HOMEWORK 4

due Monday, May 6

1. Find all values of $\log(\log i)$.
2. Let G be the domain obtained by deleting the ray $(-\infty, 0]$ from \mathbb{C} and let $f(z)$ be a branch of $\log z$ in G . Show that f maps G both one-to-one and onto a horizontal strip of the form $y_0 < \operatorname{Im} z < y_0 + 2\pi$.
3. Let $0 < \alpha < 2$. Show that, for an appropriate branch of $\arg z$, $w = z^\alpha$ maps the upper half-plane $\operatorname{Im} z > 0$ both one-to-one and onto the sector $0 < \arg w < \alpha\pi$. Show also that the boundary is carried to the boundary.

4. For the function

$$f(z) = \left(\frac{z+1}{z-1} \right)^2,$$

with $f(1) = \infty$ and $f(\infty) = 1$, find the images of (a) the extended real axis (b) the extended imaginary axis (c) the half-plane $\operatorname{Re} z > 0$.

5. Let G be the set obtained by removing from \mathbb{C} the interval $[-1, 1]$. Find the image of G under the map $z \mapsto \frac{z+1}{z-1}$. Specify a branch of $\sqrt{\frac{z+1}{z-1}}$ in G .