

HOMework 5

due Monday, May 13

1. For each $0 \leq R \leq \infty$, suggest a power series with radius of convergence R . Justify your examples.
2. Let k be a positive integer and let z_0 be a point of \mathbb{C} . Find the power series with center z_0 that represents the function z^k .
3. If the power series $\sum_{n=0}^{\infty} a_n z^n$ has radius of convergence R , what are the radii of convergence of the power series $\sum_{n=0}^{\infty} a_n z^{2n}$ and $\sum_{n=0}^{\infty} a_n^2 z^n$?
4. What function is represented by the power series $\sum_{n=1}^{\infty} n^2 z^n$?
5. Prove that the power series $\sum_{n=1}^{\infty} z^n/n$ converges at every point of the unit circle except the point $z = 1$.