Complex Analysis, Spring 2019

Grinshpan

Homework 6

due Monday, May 20

1. Let z_1 and z_2 be distinct points of $\mathbb{C}$. Evaluate the integrals

$$\int_{[z_1,\ z_2]} z^n dz \quad \text{and} \quad \int_{[z_1,\ z_2]} \overline{z}^n dz$$

for each nonnegative integer n. ($[z_1,\ z_2]$ is the directed line segment from z_1 to z_2.)

2. Let γ_1 be the semicircle from 1 to -1 through i, and let γ_2 be the semicircle from 1 to -1 through $-i$. Compute

$$\int_{\gamma_1} z^2 dz, \quad \int_{\gamma_2} z^2 dz, \quad \int_{\gamma_1} \overline{z}\, dz, \quad \text{and} \quad \int_{\gamma_2} \overline{z}\, dz.$$

3. Consider the field $F(z) = 1/z$. Find the work done by F along the circle $|z| = 1$ and the flux of F across the circle $|z| = 1$.

4. Show that $\int_\gamma e^z dz = 0$ for every smooth closed curve γ in $\mathbb{C}$.

5. Derive the formula $\frac{1}{2\pi}\int_0^{2\pi} \cos^{2n} t dt = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)}$ by integrating the function

$$\frac{1}{z}\left(z + \frac{1}{z}\right)^{2n}$$

around the unit circle.