7. Let $u$ be a given vector in the sum $U_1 + \cdots + U_n$. Prove that the sum is direct if and only if $u$ has a unique expression $u = u_1 + \cdots + u_n$ with $u_j \in U_j$ for $j = 1, \ldots, n$.

8. Prove that $\mathcal{P}(\mathbb{C})$ is not finite dimensional.

9. Suppose that $p_0, \ldots, p_n$ are polynomials over $\mathbb{F}$ such that $p_k(1) = 0$ for each $k$. Can they be linearly independent in $\mathcal{P}_n(\mathbb{F})$?

10. Let $U$ be a subspace of a space $V$ and $\dim U = \dim V$ be finite. Prove that $U = V$. What if the dimensions are infinite?

11. Let $U$ and $V$ be two subspaces of $\mathbb{R}^8$ such that $\dim U = 3, \dim V = 5$, and $U + V = \mathbb{R}^8$. Prove that $U \cap V = \{0\}$.

12. Let $U$ be the subspace of $\mathbb{R}^5$ consisting of vectors $(x_1, x_2, x_3, x_4, x_5)$ with $x_1 = 3x_2$ and $x_3 = 7x_4$. Find a basis for $U$.

13. Let $V$ be a finite dimensional direct sum of subspaces $U_1, \ldots, U_n$. Show that $\dim V = \dim U_1 + \cdots + \dim U_n$. 