HOMEWORK 3

Due Friday, February 13

Each problem is worth 10 points.

14. Let the vectors $u_1, \ldots, u_n$ span $V$ and let $T$ be a linear transformation of $V$ onto $W$. Prove that $Tu_1, \ldots, Tu_n$ span $W$.

15. Let the vectors $u_1, \ldots, u_n$ be linearly independent in $V$ and let $T$ be a one-to-one linear transformation of $V$ into $W$. Prove that $Tu_1, \ldots, Tu_n$ are linearly independent.

16. Let $U$ be a subspace of a finite dimensional space $V$. Construct a linear transformation $T$ on $V$ such that null $T = U$. Can you construct two different ones?

17. Let $U$ be a subspace of a finite dimensional space $V$. Construct a linear transformation $T$ on $V$ such that range $T = U$. Can you construct two different ones?

18. Let $S$ and $T$ be two linear transformations on a finite dimensional vector space. Prove that if $ST = I$ then $TS = I$. Here $I$ is the identity transformation.

19. Give an example of a linear transformation whose null space is the same as its range. Can you think of one on $\mathbb{R}^3$?