Due Friday, March 12

Each problem is worth 10 points.

31. Let $S$ and $T$ be two rotations of $\mathbb{R}^3$ that leave the origin unchanged. Do $S$ and $T$ have to commute? Give a proof or a counterexample.

32. Let $T$ be in $\mathcal{L}(V)$. Prove that the intersection of any collection of subspaces of $V$ invariant under $T$ is invariant under $T$.

33. Define $T$ on $\mathbb{F}^3$ by $T(a, b, c) = (2b, 0, 5c)$. Find all eigenvalues and eigenvectors of $T$.

34. Let $n$ be a positive integer and let $T$ be in $\mathcal{L}(\mathbb{F}^n)$ be defined by $T(x_1, \ldots, x_n) = (\sum_{k=1}^n x_k, \ldots, \sum_{k=1}^n x_k)$. Find all eigenvalues and eigenvectors of $T$.

35. Let $S, T$ be two commuting linear transformations on $\mathcal{L}(V)$. Prove that null $(T - \lambda I)$ is invariant under $S$ for every scalar $\lambda$.

36. Let $P$ be in $\mathcal{L}(V)$ and $P^2 = P$. Prove that $V = \text{null } P \oplus \text{range } P$.

37. Let $V = C[-\pi, \pi]$ be the vector space of continuous real-valued functions on the interval $[-\pi, \pi]$. Verify that $\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t)g(t)dt$ is an inner product on $V$.

38. Let $V$ be as in the preceding question. Prove that the functions $\frac{1}{\sqrt{2}}, \sin x, \cos x, \ldots, \sin kx, \cos kx, \ldots$ form an (infinite) orthonormal system in $V$ (Fourier basis).