Each problem is worth 10 points.

49. Let $V$ and $W$ be finite-dimensional inner-product spaces and let $T$ be in $\mathcal{L}(V,W)$. Prove that $T$ is injective if and only if $T^*$ is surjective. Prove that $T$ is surjective if and only if $T^*$ is injective.

50. Explain why there is no normal linear transformation $T$ on $\mathbb{R}^3$ such that $T(1,2,3) = (0,0,0)$ and $T(2,5,7) = (2,5,7)$.

51. Let $V$ be a finite-dimensional inner product space and let $T$ in $\mathcal{L}(V)$ be normal. Prove that null $T^k = \text{null } T$ and range $T^k = \text{range } T$ for every positive integer $k$. 