

Optimal Transport Theory and Isoperimetric Problems

Given an arbitrary solid A in space, let A^* be a sphere with the same volume as A , we can ask how to optimally transport a uniform measure on A to a uniform measure on A^* , i.e. we ask for a measurable map $T : A \rightarrow A^*$ that pushes the uniform measure on A to that on A^* and minimizes $\int_A \|x - T(x)\|^2 dx$. This is a special case of the Monge-Kantorovich problem. An argument, credited to Gromov, illustrates that the optimal transport map T can be used to prove that A^* must have a smaller surface area than A , thus proving the isoperimetric inequality. The argument is based on a characterization of T due to Brenier. I will discuss this line of ideas, as well as a related open question posted by Cédric Villani.