Problem 24, Section 3.4

An aircraft flying at 4000 ft at a rate of 300 mph passes an observer on the ground. The observer keeps track of the elevation angle $\theta = \theta(t)$ (in radians).

How fast is $\theta$ changing when $\theta = \frac{\pi}{6}$? Express the answer in degrees per second.

Solution.

Let $x = x(t)$ be the horizontal distance between the observer and the aircraft (in feet).

$$\frac{dx}{dt} = 300 \times \frac{5280}{3600} = 440 \text{ ft/s}$$

$$\theta = \frac{\pi}{2} - \arctan(x/4000)$$

$$\frac{d\theta}{dt} = -\frac{1}{(x/4000)^2 + 1} \cdot \frac{1}{4000} \cdot \frac{dx}{dt}$$

$$\left. \frac{d\theta}{dt} \right|_{\theta = \frac{\pi}{6}} = \left. -\frac{1}{(\sqrt{3})^2 + 1} \right| \frac{1}{4000} \cdot \frac{dx}{dt} = \left. -\frac{440}{4 \cdot 4000} \right| = -\frac{11}{400} = -0.0275 \text{ rad/s}$$

$$\left. \frac{d\theta}{dt} \right|_{\theta = 30^\circ} = \left. -\frac{11}{400} \times \frac{180}{\pi} \right| = \left. -\frac{99}{20\pi} \approx -1.58 \text{ deg/s} \right.$$