

## THE RULE OF THREE FOR COMMUTATION RELATIONS

ABSTRACT. In the 1970's, Lascoux and Schützenberger introduced noncommutative versions of the elementary symmetric functions and showed that they commute modulo Knuth equivalence. This result has since been generalized to other settings by Fomin-Greene, Lam, and others. We present several further remarkable generalizations of these results which have the form: if certain commutation relations hold for three or fewer variables at a time, then they hold for any number of variables. We will give “picture-proofs” for some of these results using van Kampen diagrams, a tool from geometric group theory. The necessary background on symmetric functions and group theory will be fully explained in the talk. This is joint work with Sergey Fomin.