Root Dilogarithms

R Boyer, Friday, February 12, 2016

A partition of a positive integer n is a sequence of positive integers with sum n; $p(n)$ is the number of all possible partitions of n; $p_k(n)$ counts the number of partitions of n with exactly k terms. Goh, Parry, and I studied the asymptotic behavior of the polynomials

$$F_n(z) = \sum_{k=1}^{n} p_k(n) z^k$$

suggested by a posting by Stanley. Note that $F_n(1) = p(n)$ whose asymptotics were first given by Hardy and Ramanjuan (1917)

$$p(n) \sim \frac{e^{\pi\sqrt{2n/3}}}{4n\sqrt{3}}$$

In the unit disk, the asymptotics of $F_n(z)$ have the general form as

$$a_n(z)\exp(L_k(z)n^{1/2})$$

where $L_k(z) = \sqrt{Li_2(z^k)}/k$, $k = 1, 2, 3$, and $Li_2(z) = \sum_{n=1}^{\infty} z^n/n^2$, the dilogarithm.

In this talk, I will focus on the results on these "root dilogarithms" used to study the asymptotics of $F_n(z)$ and related families.