EXAM II ANSWERS

1. A) \( y(x) = x + c_1 \cos x + c_2 \sin x \).
1. B) \( y(x) = -\frac{1}{3} \sin(2x) + c_1 \cos x + c_2 \sin x \).
1. C) \( y(x) = x - \frac{1}{3} \sin(2x) + 3 \cos x + (3 - \frac{\pi}{2}) \sin x \).

2. A) \( y'' - y' = 0 \) has the general solution \( y(x) = c_1 e^x + c_2 \).
2. B) \( 2r^2 + \alpha r + 2 = 0 \) must have the double root \( r = 1 \). So \( \alpha = -4 \).
2. C) \( x(t) = c_1 e^{-t} + c_2 + c_3 t + e^t [c_4 + c_5 t] + c_6 \cos(2t) + c_7 \sin(2t) \\
+ e^{-2t} [(c_8 + c_9 t + c_{10} t^2) \cos(2t) + (c_{11} + c_{12} t + c_{13} t^2) \sin(2t)] \).

3. A) The Wronskian of \( e^t \sin t \) and \( e^t \cos t \) is \( W(t) = -e^{2t} \).
3. B) \( f(t) \) and \( g(t) \) are linearly independent on any interval.
3. C) \( f(t)g(t) - f(t)g(t) = W(t) = \dot{i} = 1 \).

4. A) \( x(t) = e^{t/2} (c_1 + c_2 t) \).
4. B) Let \( x_p = ue^{t/2} + vte^{t/2} \) where \( \dot{u} e^{t/2} + \dot{v} te^{t/2} = 0 \) and \\
\( \dot{u} \frac{1}{2} e^{t/2} + \dot{v} (1 + \frac{1}{2} t) e^{t/2} = 4 e^{t/2} \). Then \( \dot{v} = 4 \) and \( \dot{u} = -4t \).
So \( x_p = (-2t^2 + 4t^2) e^{t/2} = 2t^2 e^{t/2} \) is a particular solution and \( x = (c_1 + c_2 t + 2t^2) e^{t/2} \) is the general solution.