Let us calculate the integral
\[
\frac{1}{\pi} \int_{-\pi}^{\pi} \frac{x}{2} \sin(kx) \, dx, \quad k = 1, 2, 3, \ldots.
\]
Integration by parts with \( u = x \), \( dv = \sin(kx) \, dx \) gives:
\[
\frac{1}{\pi} \int_{-\pi}^{\pi} \frac{x}{2} \sin(kx) \, dx = -\frac{1}{2\pi k} x \cos(kx) \bigg|_{-\pi}^{\pi} + \frac{1}{2\pi k} \int_{-\pi}^{\pi} \cos(kx) \, dx
\]
\[
= -(-1)^k \frac{1}{k} + \frac{1}{2\pi k^2} \sin(kx) \bigg|_{-\pi}^{\pi}
\]
\[
= (-1)^{k+1} \frac{1}{k}.
\]

Interestingly enough, the above calculation tells us “how much of \( \sin(kx) \) is contained in \( x/2 \)”. So
\[
\frac{x}{2} = \sin x - \frac{1}{2} \sin(2x) + \frac{1}{3} \sin(3x) - \ldots, \quad -\pi < x < \pi.
\]
Here is some supporting evidence:

\[
y = x/2
\]
\[
y = \sin x - \frac{1}{2} \sin(2x) + \ldots - \frac{1}{100} \sin(100x)
\]