There are three most common inverse trigonometric functions: inverse sine, inverse cosine, and inverse tangent.

1. INVERSE SINE FUNCTION

The inverse sine function is written as \( y = \sin^{-1}(x) \) or \( y = \arcsin x \).
(Not to be confused with \( y = 1 / \sin x \).)

The domain of \( \arcsin x \) is the interval \([-1, 1]\) and its range is \([-\pi/2, \pi/2]\).

For any number \( x \) between \(-1\) and \(1\), \( \arcsin x \) is the angle between \(-\pi/2\) and \(\pi/2\) whose sine is \(x\).

In other words, let \(-1 \leq x \leq 1\) and let \(-\pi/2 \leq \theta \leq \pi/2\). Then to say that 

\[ \arcsin x = \theta \]

is the same as to say that 

\[ \sin \theta = x. \]

Here are some examples:

\[
\begin{align*}
\arcsin 0 &= 0 & \text{because} & \sin 0 &= 0 \\
\arcsin 1 &= \frac{\pi}{2} & \text{because} & \sin \frac{\pi}{2} &= 1 \\
\arcsin(\frac{-\sqrt{3}}{2}) &= \frac{-\pi}{3} & \text{because} & \sin(\frac{-\pi}{3}) &= \frac{-\sqrt{3}}{2} \\
\arcsin 1 &= \frac{\pi}{2} & \text{because} & \sin \frac{\pi}{2} &= 1.
\end{align*}
\]

By the definition of \( \arcsin \) we have

\[ \arcsin(\sin \theta) = \theta, \quad \text{for any } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \]

and 

\[ \sin(\arcsin x) = x, \quad \text{for any } -1 \leq x \leq 1. \]

More examples. First, \( \sin \pi = 0 \) but \( \arcsin 0 \neq \pi \) because \( \pi \) does not belong to \([-\pi/2, \pi/2]\). Second, \( \arcsin \frac{\pi}{3} \) is not defined because \( \frac{\pi}{3} \) does not belong to \([-1, 1]\).
2. Inverse cosine function

The inverse cosine function is written as \( y = \cos^{-1}(x) \) or \( y = \arccos x \).
(Not to be confused with \( y = 1/\cos x \).)

The domain of \( \arccos x \) is the interval \([-1, 1]\) and its range is \([0, \pi]\).

**For any number \( x \) between \(-1\) and \(1\), \( \arccos x \) is the angle between \(0\) and \(\pi\) whose cosine is \(x\).**

In other words, let \(-1 \leq x \leq 1\) and let \(0 \leq \theta \leq \pi\). Then to say that

\[ \arccos x = \theta \]

is the same as to say that

\[ \cos \theta = x . \]

Here are some examples:

\begin{align*}
\arccos 0 &= \frac{\pi}{2} \quad \text{because} \quad \cos \frac{\pi}{2} = 0 \\
\arccos \frac{1}{2} &= \frac{\pi}{3} \quad \text{because} \quad \cos \frac{\pi}{3} = \frac{1}{2} \\
\arccos \left( -\frac{\sqrt{3}}{2} \right) &= \frac{5\pi}{6} \quad \text{because} \quad \cos \left( \frac{5\pi}{6} \right) = -\frac{\sqrt{3}}{2} \\
\arccos 1 &= 0 \quad \text{because} \quad \cos 0 = 1.
\end{align*}

By the definition of \( \arccos \) we have

\[ \arccos(\cos \theta) = \theta, \quad \text{for any} \quad 0 \leq \theta \leq \pi, \]

and

\[ \cos(\arccos x) = x, \quad \text{for any} \quad -1 \leq x \leq 1. \]

More examples. First, \( \cos(2\pi) = 1 \) but \( \arccos 1 \neq 2\pi \) because \(2\pi\) does not belong to \([0, \pi]\). Second, \( \arccos 5 \) is not defined because 5 does not belong to \([-1, 1]\).
3. Inverse Tangent Function

The inverse tangent function is written as \( y = \tan^{-1}(x) \) or \( y = \arctan x \). (Not to be confused with \( y = 1/\tan x = \cot x \).)

The domain of \( \arctan x \) is the entire real line \(( -\infty, \infty )\) and its range is \(( -\frac{\pi}{2}, \frac{\pi}{2} )\).

For any number \( x \), \( \arctan x \) is the angle between \(-\frac{\pi}{2}\) and \( \frac{\pi}{2}\) whose tangent is \( x \).

In other words, let \(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\). Then to say that
\[
\arctan x = \theta
\]
is the same as to say that
\[
\tan \theta = x.
\]

Here are some examples:

\[
\begin{align*}
\arctan 0 &= 0 & \text{because } \tan 0 &= 0 \\
\arctan 1 &= \frac{\pi}{4} & \text{because } \tan \frac{\pi}{4} &= 1 \\
\arctan(-\sqrt{3}) &= -\frac{\pi}{3} & \text{because } \tan(-\frac{\pi}{3}) &= -\sqrt{3} \\
\arctan \frac{1}{\sqrt{3}} &= \frac{\pi}{6} & \text{because } \tan \frac{\pi}{6} &= \frac{1}{\sqrt{3}}
\end{align*}
\]

By the definition of \( \arctan \) we have
\[
\arctan(\tan \theta) = \theta, \quad \text{for any } -\frac{\pi}{2} < \theta < \frac{\pi}{2},
\]
and
\[
\tan(\arctan x) = x, \quad \text{for any } x.
\]

More examples. First, \( \tan \pi = 0 \) but \( \arctan 0 \neq \pi \) because \( \pi \) does not belong to \(( -\frac{\pi}{2}, \frac{\pi}{2} )\). Second, \( \arctan x \) is defined for any \( x \).
4. LESS COMMON INVERSE TRIGONOMETRIC FUNCTIONS

We can also consider inverse cotangent, inverse secant, and inverse cosecant. These functions will not appear so often in the course and the information below is just for your reference.

**Inverse cotangent.** Written as \( y = \cot^{-1} x \) or \( y = \arccot(x) \). Given a number \( -\infty < x < \infty \), \( \arccot(x) \) returns the angle \( 0 < \theta < \pi \) whose cotangent is \( x \). For instance, \( \arccot(0) = \frac{\pi}{2} \), \( \arccot(1) = \frac{\pi}{4} \), \( \arccot(-1) = \frac{3\pi}{4} \), \( \arccot(-\sqrt{3}) = \frac{5\pi}{6} \).

**Inverse secant.** Written as \( y = \sec^{-1} x \) or \( y = \text{arcsec}(x) \). Given a number \( x \) with \( |x| \geq 1 \), \( \text{arcsec}(x) \) returns the angle \( 0 \leq \theta \leq \pi \), \( \theta \neq \frac{\pi}{2} \), whose secant is \( x \). The reason why \( \frac{\pi}{2} \) is not in the range is that \( \sec \frac{\pi}{2} \) is undefined. Examples: \( \text{arcsec}(1) = 0 \), \( \text{arcsec}(2) = \frac{\pi}{3} \), \( \text{arcsec}(-1) = \pi \), \( \text{arcsec}(-\frac{2}{\sqrt{3}}) = \frac{5\pi}{6} \).

**Inverse cosecant.** Written as \( y = \csc^{-1} x \) or \( y = \text{arccsc}(x) \). Given a number \( x \) with \( |x| \geq 1 \), \( \text{arccsc}(x) \) returns the angle \( -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \), \( \theta \neq 0 \), whose cosecant is \( x \). The reason why 0 is not in the range is that \( \csc 0 \) is undefined. Examples: \( \text{arccsc}(1) = \frac{\pi}{2} \), \( \text{arccsc}(2) = \frac{\pi}{3} \), \( \text{arccsc}(-1) = -\frac{\pi}{2} \), \( \text{arccsc}(-\frac{2}{\sqrt{3}}) = -\frac{\pi}{3} \).