Drexel Analysis Seminar

May 13, 2019

2-3 PM, Korman 245

Speaker: Joshua Jackson (Drexel)

Title: A Determinantal Representation for Bivariate Polynomials whose Bezoutians admit a Canonical Factorization.

Abstract: We prove that for every bivariate polynomial $p(z_1, z_2)$ of bidegree (n_1, n_2) with $p(0,0) = 1$ and $p(z_1, z_2) \neq 0$ on $\mathbb{T}^2$, if the Bezoutian $Bez(p_{z_1}, \bar{p}_{\frac{1}{\bar{z}_1}})$ admits a canonical factorization, then $p(z_1, z_2)$ admits a determinantal representation

$$p(z_1, z_2) = \det(I_{n_1+n_2} - KZ),$$

where $Z = z_1 I_{n_1} \oplus z_2 I_{n_2}$, and K is a $\begin{pmatrix} I_{n_1} & 0 \\ 0 & J \end{pmatrix}$-cocontraction with $J = \begin{pmatrix} I_{n_2-k} & 0 \\ 0 & -I_k \end{pmatrix}$ and k is the number of roots of $p(1, z_2)$ inside $\mathbb{D}$.

The $k = 0$ case of this result is due to Kummert (1990). In this case the Bezoutian is positive definite. The proof was streamlined by Grinshpan, Kaliuzhnyi-Verbovetskyi, Vinnikov, and Woerdeman (2013). The present result represents a generalization to the case where the Bezoutian is allowed to be indefinite.