LONG-RESOLVENT REPRESENTATIONS OF RATIONAL CAYLEY INNER HERGLOTZ-AGLER FUNCTIONS

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Abstract. The Bessmertnyĭ class consists of rational matrix-valued functions of d complex variables which have a *long-resolvent representation*, i.e., which are representable as the Schur complement of a block of a linear pencil $A(z) = z_1A_1 + \cdots + z_dA_d$ whose coefficients A_k are positive semidefinite matrices. It coincides with the subclass of rational functions in the Herglotz–Agler class over the right poly-halfplane which are homogeneous of degree one and which are Cayley inner. The latter means that such a function is holomorphic on the right poly-halfplane and takes skew-Hermitian matrix values on $(i\mathbb{R})^d$, or equivalently, is the double Cayley transform (over the variables and over the matrix values) of an inner function on the unit polydisk. Using Agler–Knese's characterization of rational inner Schur–Agler functions on the polydisk, extended now to the matrix-valued case, and applying appropriate Cayley transformations, we extend Bessmertnyĭ's representation to all rational Cayley inner Herglotz–Agler functions on the right poly-halfplane, where a linear pencil $A(z)$ is now in the form $A(z) = A_0 + z_1A_1 + \cdots + z_dA_d$ with A_0 skew-Hermitian and the other coefficients A_k positive semidefinite matrices. The talk is based on a recent joint work with J. A. Ball (Virginia Tech).