Drexel Analysis Seminar

January 8, 2016

2-2:50 PM, Korman 245

Speaker: David Kimsey (Newcastle University, UK)

Title: Moment problems on $\mathbb{R}^{\mathbb{N}}$

Abstract: Let $\mathbb{R}^{\mathbb{N}}$ denote the metric space of countably infinite tuples of real numbers endowed with the Tychonoff topology. Given a closed set $K \subseteq \mathbb{R}^{\mathbb{N}}$ and a mapping $s : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$, we will prove that

$$s_{m_1,m_2,\ldots} = \int \cdots \int_{\mathbb{R}^{\mathbb{N}}} \prod_{d=1}^{\infty} x_d^{m_d} d\mu(x_1, x_2, \ldots),$$

where $\{m_d\}_{d=1}^{\infty}$ are nonnegative integers with $m_d = 0$ for all but finitely many $d \in \mathbb{N}$, if and only if a natural analogue of the Riesz-Haviland functional is K-positive, i.e.,

$$p|_K \geq 0 \Longrightarrow \mathcal{L}_s(p) \geq 0,$$

where $p(x) = \sum x^m a_m$ is a polynomial on $\mathbb{R}^{\mathbb{N}}$ and $\mathcal{L}_s(p) = \sum a_m s_m$. The proof will rely on the classical solution of the K-moment problem on $\mathbb{R}^d$ due to E. K. Haviland. We will also highlight analogues in $\mathbb{R}^{\mathbb{N}}$ of Carleman's criterion for determinateness and Schmüdgen's moment problem solution when K is a compact semi-algebraic set.

This talk is based on joint work with Daniel Alpay and Palle Jorgensen.