Drexel Analysis Seminar

March 1, 2013

3-3:50 PM, Korman 245

Speaker: Andrey Melnikov

Title: Proof of existence of a local solution of a KdV equation on the line with analytic initial potential.

Abstract: In this talk I will present a proof of a new result, hoping for a help from the audience for proofreading and clearance of presentation. The problem that I present is the initial–value problem for the KdV equation: the initial value is given on the real line and is assumed to be analytic function. It will be shown that for each x there exists an interval $[0, T_x]$, such that a solution of KdV exists for t in $[0, T_x]$. The solution will coincide with the given analytic potential for $t = 0$.

Unfortunately, it can't be shown at this time that a global solution exists for all x in $\mathbb{R}$ and all t in $[0, T_0]$ for a fixed T_0, because theoretically it might happen that T_x, mentioned above goes to zero when x approaches infinity.

I will use Krein spaces, unbounded operators of special kind between them, Stieltjes moment problem, "analytic" measures on the line, an extended notion of a node, realizations of matrix-valued functions. Finally, we will see the notion of the tau function and its significance for the existence of a solution.

Hope to see you all, because I am not going to sketch a result, but rather to obtain your approve that all notions and theorems work and that I indeed solved this very important problem.

Before me, people solved this problem for initial value:
1. in Schwartz class,
2. Integrable, decreasing fast enough,
3. Periodic,
4. Almost periodic (with some Diophantine assumptions on the coefficients of cos's, sin's).

And it seems that using this new idea of a vessel it
1. can be extended (using approximations by analytic functions) to just locally integrable initial conditions.
2. can be copied to show the existence of solutions of evolutionary NLS, Boussinesq equations, which have also been shown to possess vessel solutions.