A) If \( x(t) = te^t \) is a solution of \( 2\ddot{x}(t) + \alpha \dot{x}(t) + 2x(t) = 0 \) then \( r^2 + \frac{\alpha}{2}r + 1 = 0 \) has a double root \( r = 1 \). Hence \( \alpha/2 = -2 \) or \( \alpha = -4 \).

B) The characteristic equation for \( \ddot{x} + 2\dot{x} + 3 = 0 \) is \( r^2 + 2r + 3 = 0 \). Its roots are \( r = -1 \pm i\sqrt{2} \). Hence \( \lambda = -1, \mu = \sqrt{2}, \) and

\[
x = e^{-t}[c_1 \cos(t\sqrt{2}) + c_2 \sin(t\sqrt{2})]
\]

is the general solution.

To take care of the initial conditions \( x(\pi/4) = 1, \dot{x}(\pi/4) = 0 \) write \( y(t) = x(t + \pi/4) \). Then \( y(t) \) satisfies the same equation (check!) and \( y(0) = x(\pi/4) = 1, \dot{y}(0) = \dot{x}(\pi/4) = 0 \). We have

\[
y = e^{-t}[A \cos(t\sqrt{2}) + B \sin(t\sqrt{2})],
\]

hence

\[
e^t y = A \cos(t\sqrt{2}) + B \sin(t\sqrt{2})
\]

\[
e^t y + e^t \dot{y} = -A\sqrt{2} \sin(t\sqrt{2}) + B\sqrt{2} \cos(t\sqrt{2}).
\]

Setting \( t = 0 \) we obtain a simple linear system for \( A \) and \( B \):

\[
\begin{align*}
A \cdot 1 + B \cdot 0 &= 1 \\
- A\sqrt{2} \cdot 0 + B\sqrt{2} \cdot 1 &= 1.
\end{align*}
\]

Hence

\[
A = 1, \quad B = 1/\sqrt{2},
\]

and

\[
y(t) = e^{-t} \left[ \cos(t\sqrt{2}) + \sin(t\sqrt{2})/\sqrt{2} \right].
\]

The last step is to recover \( x(t) = y(t - \pi/4) \),

\[
x = e^{-t+\pi/4} \left[ \cos((t - \pi/4)\sqrt{2}) + \sin((t - \pi/4)\sqrt{2})/\sqrt{2} \right].
\]

The intended question, \( \ddot{x} + 2\dot{x} + 2 = 0, x(\pi/4) = 1, \dot{x}(\pi/4) = 0 \).

The roots of the indicial equation \( r^2 + 2r + 2 = (r + 1)^2 + 1 = 0 \) are \( r = -1 \pm i \). Hence \( x = e^{-t}(a \cos t + b \sin t) \) is the general solution.

Notice that \( \dot{x} = e^{-t}[(b - a) \cos t - (b + a) \sin t] \). So if \( \dot{x}(\pi/4) = 0 \) then \( b - a = b + a \), which means that \( a = 0 \). To match \( x(\pi/4) = 1 \) take \( b = e^{\frac{\pi}{4}}\sqrt{2} \),

\[
x = \sqrt{2} e^{-t+\frac{\pi}{4}} \sin t.
\]