1. A) First order. B) Nonlinear. C) \( y' = \frac{1}{k} \) along \( y = k \).
D) No, \( y \) is increasing if positive.
E) \( yy' = 1 \)
\[ y^2/2 = x + c \]
\[ y^2 = 2x + c \quad \text{and} \quad y(0) = 1, \]
\[ y = \sqrt{2x + 1}. \]
F) The solution is defined for \( x > -\frac{1}{2} \). Exclude \( x = -\frac{1}{2} \) because a solution must be continuously differentiable.

2. A) Separation of variables.
Observe that \( y = 2 \) is a solution. If \( y \neq 2 \) then
\[ \frac{dy}{y - 2} = -dx \]
\[ \ln |y - 2| = -x + c \]
\[ |y - 2| = ce^{-x} \]
\[ y = 2 + ce^{-x}. \]
B) Integrating factor.
\[ y' + y = 2 \]
\[ e^x y' + e^x y = 2e^x \]
\[ (e^x y)' = 2e^x \]
\[ e^x y = 2e^x + c \]
\[ y = 2 + ce^{-x}. \]
C) Undetermined coefficients.
If \( y_0' = -y_0 \) then \( y_0 = ce^{-x} \). Look for \( y \) in the form \( y = y_0 + a, \) a constant:
\[ (y_0 + a)' = -(y_0 + a) + 2 \]
\[ y_0' = -y_0 - a + 2 \]
\[ a = 2. \]
Hence \( y = ce^{-x} + 2. \)
C*) Variation of parameters.
If \( y_0' = -y_0 \) then \( y_0 = ce^{-x} \). Look for \( y \) in the form \( y = c(x)e^{-x}: \)
\[ (c(x)e^{-x})' = -c(x)e^{-x} + 2 \]
\[ c'(x)e^{-x} - c(x)e^{-x} = -c(x)e^{-x} + 2 \]
\[ c'(x)e^{-x} = 2 \]
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\[ c'(x) = 2e^x \]
\[ c(x) = 2e^x + k. \]

Hence \( y = 2 + ke^{-x}. \)

3. A) Two equilibrium solutions: \( x = 0 \) (unstable) and \( x = 2 \) (stable).
B) If \( x(0) = 1 \) then \( x(t) \neq 0. \) Divide the equation by \( -x^2: \)

\[ -x^{-2}\dot{x} = -2/x + 1. \]

If \( u = 1/x \) (Bernoulli substitution) then

\[ \dot{u} = -2u + 1 \]
\[ u = \frac{1}{2} + ce^{-2t} \]
\[ x = \frac{2e^{2t}}{e^{2t} + c} \]
\[ x = \frac{2e^{2t}}{e^{2t} + 1}, \] because \( x(0) = 1. \)

4. A) Not exact: \( \frac{\partial}{\partial y} (2y - e^x) = 2 \neq 1 = \frac{\partial}{\partial x} x. \)
B) Integrating factor depending on \( x \) only: \( \mu = x \)
C) Multiply by \( x: (2xy - x^2)dx + x^2dy = 0. \) Look for the potential function \( F(x, y) \) such that \( \frac{\partial}{\partial x} F = 2xy - xe^x \) and \( \frac{\partial}{\partial y} F = x^2. \) Integration gives \( F = x^2y - xe^x + e^x. \) Hence \( x^2y - xe^x + e^x = c \) or \( y = \frac{c + e^x(x-1)}{x^2}, \, x \neq 0. \)