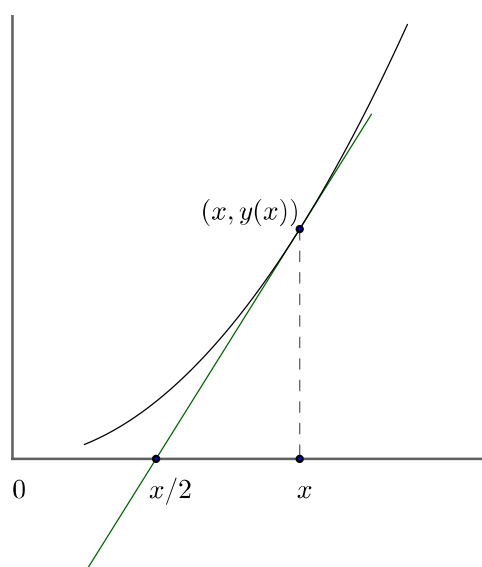


AN EXAMPLE ON SEPARATION OF VARIABLES

Suppose that an arc $y = y(x)$ in the first quadrant has the property that, for each x , the tangent line at x meets the horizontal axis halfway, at $\frac{x}{2}$.



Can you identify the arc?

The plot above suggests the following relation: $y'(x) = \frac{y(x)}{x/2}$.

We will solve this differential equation using separation of variables:

$$\begin{aligned}\frac{dy}{dx} &= 2 \frac{y}{x} \\ \frac{dy}{y} &= 2 \frac{dx}{x} \\ \int \frac{dy}{y} &= 2 \int \frac{dx}{x} \\ \ln y &= 2 \ln x + C \\ \ln y &= \ln(x^2) + C \\ y &= Ax^2 \quad (A = e^C)\end{aligned}$$

So the arc is a parabola.