The Normal Defect of Some Classes of Matrices

Abstract: An $n \times n$ matrix A has a normal defect of k if there exists an $(n+k)\times(n+k)$ normal matrix A_{ext} with A as a leading principal submatrix and k minimal. In this paper we compute the normal defect of a special class of 4×4 matrices, namely matrices whose only nonzero entries lie on the superdiagonal, and we provide details for constructing minimal normal completion matrices A_{ext}. We also prove a result for a related class of $n \times n$ matrices. Finally, we present an example of a 6×6 block diagonal matrix having the property that its normal defect is strictly less than the sum of the normal defects of each of its blocks, and we provide sufficient conditions for when the normal defect of a block diagonal matrix is equal to the sum of the normal defects of each of its blocks.