Complex Analysis

Grinshpan

Worksheet on subsets of $\mathbb{C}$

Describe and plot the subset of the complex plane defined by the given equation.
Decide whether the subset is open, closed, or neither open nor closed.
Indicate the interior, the boundary, and the closure of the subset.
Decide whether the subset is connected and whether it is convex.

1. $|z| = 1$ $|z| \leq 1$ $|z+1| = 2$ $1 < |z| < 2$ $1 < |z+1| < 2$

$|z| < 1$ $|z| > 1$ $|z+1| < 2$ $|z| > 0$ $|z-1| = 2|z+1|$

2. $\operatorname{Re} z = 3$ $\operatorname{Re} z > 3$ $0 \leq \operatorname{Re} z \leq 1$ $\operatorname{Re}(z^2) = 0$ $\operatorname{Im}(z^2) = 0$

$\operatorname{Im} z = -1$ $\operatorname{Re}(iz+1) > 0$ $|z+1| > |z-1|$ $\operatorname{Re}(z^2) = 1$ $\operatorname{Im}(z^2) > 1$

3. $\arg z = 0$ $0 \leq \arg z \leq \dfrac{\pi}{4}$ $\arg(z^3) = 0$ $\arg(z^{10}) = 0$ $\arg(z^{10} - 1) = 0$

$\arg z = \pi$ $\arg(iz) = 0$ $\arg(z+1) = \dfrac{\pi}{2}$ $\arg(z^2 - 1) = 0$ $\arg \dfrac{1+z}{1-z} = 0$

4. $z^{10} - 1 = 0$ $|z-1| > 2|z+1|$ $|z^2 - 1| = 1$ $|z^2 - 1| = \dfrac{1}{2}$ $\left|z + \dfrac{1}{z}\right| = 1$