Mapping $z \mapsto 1/z$

The linear-fractional transformation $\varphi(z) = 1/z$ is induced by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

This is a one-to-one mapping of $\overline{\mathbb{C}}$, with $\varphi(0) = \infty$, $\varphi(\infty) = 0$, and $\varphi \circ \varphi = \text{id}$.

The fixed points of φ are $z = \pm 1$.

For finite $z \neq 0$, the mapping is holomorphic, $\varphi'(z) = -1/z^2$, and conformal,

$\arg \varphi(z) = -\arg z$ and $|\varphi(z)| = 1/|z|$. Note that $\arg \varphi'(z) = \pi + 2 \arg z$ and $|\varphi'(z)| = 1/|z|^2$.

The unit disk $|z| < 1$ is mapped onto its exterior domain $|z| > 1$.

The unit circle $|z| = 1$ remains invariant: $\varphi(z) = \overline{z}/|z|^2 = \overline{z}$.

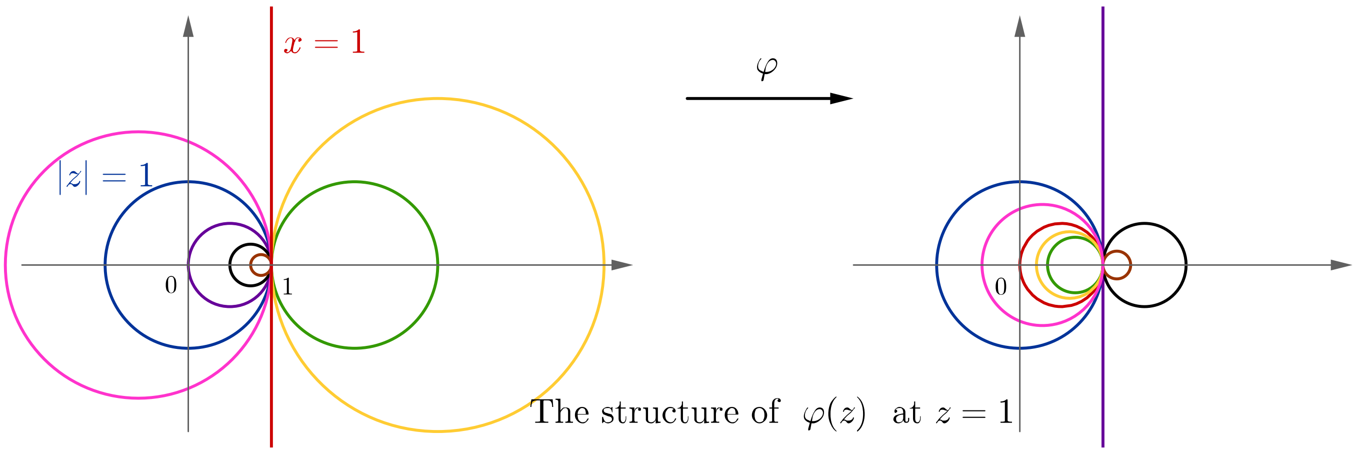
For fixed θ, the ray $\arg z = \theta$ is mapped onto the ray $\arg z = -\theta$ (direction reversed).

For fixed r, the circle $|z| = r$ is mapped onto the circle $|z| = 1/r$ (direction reversed).

For $x_0 \neq 0$, the line $\operatorname{Re} z = x_0$ is mapped onto the circle $|z - \frac{1}{2x_0}| = \frac{1}{2|x_0|}$.

For $y_0 \neq 0$, the line $\operatorname{Im} z = y_0$ is mapped onto the circle $|z + \frac{i}{2y_0}| = \frac{1}{2|y_0|}$.

For $x_0 \neq 0, 1$, the circle $|z - x_0| = |1 - x_0|$ is mapped onto the circle $|z - \frac{1+1/x_0}{2}| = \frac{|1-1/x_0|}{2}$.



Which circle is mapped onto the line $x = 1$?

Which line is mapped onto the circle $|z - 1/2| = 1/2$?

Shade the image of the crescent formed by the circles $|z| = 1$ and $|z - 1/2| = 1/2$.