The derivatives of $x^x$, $(x^x)^x$, and $x^{(x^x)}$.

The function $y = x^x$ is defined for $x > 0$. Its derivative can be found by logarithmic differentiation:

$$y = x^x$$
$$\ln y = x \ln x$$
$$y'/y = \ln x + x/x$$
$$y' = x^x(\ln x + 1).$$

The functions $y = (x^x)^x = x^{(x^x)}$ and $y = x^{(x^x)}$ are not the same. To see this, compare $(3^3)^3 = 3^9$ and $3^{(3^3)} = 3^{27}$.

Their derivatives can be found by the same method:

$$y = x^{(x^x)}$$
$$\ln y = x^x \ln x$$
$$y'/y = (x^x)' \ln x + x^{(x^x)}/x$$
$$y' = x^x (\ln x + 1) \ln x + x^{x-1}$$
$$y' = x^{x^x} (x \ln x (\ln x + 1) + 1).$$

By convention, $x^{x^x} = x^{(x^x)}$.

For large values of $x$, $x^{x^x}$ leaves $x^{x^2}$ far behind. The two functions agree at $x = 1$ and at $x = 2$. Near the origin, the graphs are as follows:

At $x = 1$, the graphs share a common tangent line. Can you find its equation?